8 Data Handling

8.1 Tables and Timetables

This section is concerned with the use of tables and timetables.

Worked Example 1

This timetable is part of a rail timetable for trains from the south west of England.

Saturdays

Penzance	**0715**	—	—	—	—	**0750**	—	**0846**
Plymouth	**0912**	—	**0935**	—	—	**1000**	—	**1035**
Ivybridge	—	—	—	—	—	**1014**	—	—
Totnes	—	—	**1002**	—	—	**1030**	—	—
Paignton	—	**0940**	—	**1001**	**1017**	—	**1030**	—
Torquay	—	**0946**	—	**1008**	**1022**	—	**1040**	—
Torre	—	—	—	—	**1025**	—	—	—
Newton Abbot	**0948**	**0958**	**1015**	**1020**	**1035**	**1044**	**1052**	**1112**
Teignmouth	—	**1005**	**1021**	**1026**	**1042**	—	**1101**	—
Dawlish	—	**1010**	**1026**	**1033**	**1047**	—	**1108**	—
Dawlish Warren	—	—	—	**1040**	**1051**	—	**1114**	—
Starcross	—	—	—	—	**1054**	—	—	—
Exeter St. Thomas	—	—	—	—	**1103**	—	—	—
Exeter St. David's	**1007**	**1023**	**1038**	**1052**	**1107**	—	**1127**	**1131**
Tiverton Parkway	—	—	**1055**	—	—	—	—	**1148**
Taunton	**1037**	**1054**	**1108**	**1118**	—	—	**1207**	**1201**
Bristol Temple Meads	**1115**	**1154**	—	**1158**	—	—	**1255**	—
London Paddington	—	—	**1325**	—		—	—	**1410**

(a) Stewart wants to travel from Penzance to London Paddington. Describe the different options shown on this timetable.

(b) How can John get from Plymouth to Bristol Temple Meads?

(c) How long does the 0715 train take to go from Penzance to Newton Abbot?

Does the 0846 train take the same time?

Solution

(a) There are several possible options, including:

(i) leave Penzance at 0715 and arrive at Newton Abbot at 0948. Then leave Newton Abbot at 1015 and arrive at London Paddington at 1325;

(ii) leave Penzance at 0846 and travel direct to London Paddington, arriving at 1410.

(b) Possible options include:

(i) leave Plymouth at 0912 and travel direct to Bristol Temple Meads, arriving at 1115;

(ii) leave Plymouth at 0935 and travel to Newton Abbot arriving at 1015. Then leave Newton Abbot at 1020 and travel to Bristol Temple Meads, arriving at 1158;

(iii) leave Plymouth at 1000 and travel to Newton Abbot, arriving at 1044. Then leave Newton Abbot at 1052 and travel to Bristol Temple Meads, arriving at 1255.

(c) The 0715 train takes 2 hours 33 minutes; the 0846 train is quicker, taking 2 hours 26 minutes.

Worked Example 2

The chart can be used to find the mileage between some Scottish towns and cities.

Aberdeen	Edinburgh	Fort William	Glasgow	Inverness	Perth	Stranraer
115						
152	**130**					
142	**44**	**103**				
104	**156**	**66**	**169**			
81	**42**	**103**	**61**	**115**		
226	**123**	**183**	**84**	**249**	**145**	

Find the distances between:

(a) Fort William and Perth,

(b) Edinburgh and Stranraer.

(c) Which two places are furthest apart?

Solution

(a) To find the distance between Fort William and Perth look in the square where the two highlighted lines meet. So the distance is 103 miles.

(b)

Aberdeen	Edinburgh	Fort William	Glasgow	Inverness	Perth	Stranraer
115						
152	**130**					
142	**44**	**103**				
104	**156**	**66**	**169**			
81	**42**	**103**	**61**	**115**		
226	**123**	**183**	**84**	**249**	**145**	

Using the same approach for Edinburgh and Stranraer gives 123 miles.

Aberdeen	Edinburgh	Fort William	Glasgow	Inverness	Perth	Stranraer
115						
152	**130**					
142	**44**	**103**				
104	**156**	**66**	**169**			
81	**42**	**103**	**61**	**115**		
226	**123**	**183**	**84**	**249**	**145**	

(c)

The largest number in the table is 249.

Using the highlight, this is the distance between Inverness and Stranraer. So these two places are furthest apart.

Aberdeen	Edinburgh	Fort William	Glasgow	Inverness	Perth	Stranraer
115						
152	**130**					
142	**44**	**103**				
104	**156**	**66**	**169**			
81	**42**	**103**	**61**	**115**		
226	**123**	**183**	**84**	**249**	**145**	

Exercises

1. Mike lives in Paignton and works in Exeter, close to the Central Station. Use the following timetable to answer these questions.

   (a) He starts work at 9.00 am. Which train should he catch?

   (b) One day he misses his normal train. What is the earliest time he can get to Exeter Central?

   (c) Mike can walk from Exeter St. David's to his office in 15 minutes. What should he do if he misses his usual train?

   (d) Sometimes Mike has to go to Bristol or London. How can he get to these places as early as possible and at what time does he arrive?

Mondays to Fridays

Penzance	—	—	—	—	—	—	0641	—
Plymouth	—	0725	0735	—	—	0818	0832	—
Ivybridge	—	—	—	—	—	0832	—	—
Totnes	—	0752	0802	—	—	0848	—	—
Paignton	0723	—	—	0800	0826	—	—	0856
Torquay	0728	—	—	0805	0832	—	—	0901
Torre	0731	—	—	0808	—	—	—	0904
Newton Abbot	0740	0805	0815	0818	0844	0900	0908	0914
Teignmouth	0747	—	—	0825	—	—	—	0921
Dawlish	0752	—	—	0830	—	—	—	0926
Dawlish Warren	0756	—	—	—	—	—	—	0929
Starcross	0759	—	—	0835	—	—	—	0933
Exeter St. Thomas	0808	—	—	0844	—	—	—	0942
Exeter St. David's	0812	0824	0834	0846	0903	—	0930	0945
Exeter Central	0818	0836	0853	0853	0923	—	0940	0953
Exmouth	0848	—	0918	0918	0948	—	—	1018
Barnstaple	—	—	—	0955	—	—	—	—
Tiverton Parkway	—	0841	0851	—	0920	—	—	—
Taunton	—	0854	0904	—	0933	—	0959	—
Bristol Temple Meads	—	0932	—	—	1009	—	1049	—
London Paddington	—	—	1110		—	—	—	—

2. Use the timetable below to answer these questions about James' journey.

   (a) James catches the 1927 at Reading. What time does this arrive at Cardiff Central?

   (b) How long does his journey take?

   (c) He wanted to arrive at Cardiff before 11.00 pm. Could he have caught a later train?

   (d) What is the latest train he could have caught from Reading to arrive at Cardiff before 11.00 pm?

London → Bristol Parkway - Cardiff - Swansea
Bath - Bristol Temple Meads

Mondays to Fridays continuation

London Paddington	1815	1830	1845	1900	1915	2000	2015	2100	2115
Reading	1840	1857	1910	1927	1942	2027	2043	2127	2145
Didcot Parkway	—	1913	1925	—	1957	—	2058	—	2203
Swindon	1910	1934	1946	1959	2018	2059	2119	2200	2224
Chippenham	1923	—	1959	—	2032	—	2133	—	2238
Bath Spa	1938	—	2012	—	2044	—	2146	—	2251
Bristol Parkway	—	2002	—	2027	—	2127	—	2228	—
Bristol Temple Meads	1956	—	2025	—	2100	—	2201	—	2306
Weston-super-Mare	2023	—	—	—	2140	—	2235	—	—
Newport	—	2024	—	2049	—	2149	—	2250	—
Cardiff Central	—	2041	—	2106	—	2206	—	2312	—
Bridgend	—	2101	—	2126	—	2226	—	2332	—
Port Talbot Parkway	—	2112	—	2137	—	2237	—	2343	—
Neath	—	2120	—	2145	—	2245	—	2351	—
Swansea	—	2135	—	2200	—	2300	—	0005	—

3. Farharaz arrives by taxi at London Paddington at 0800. Use the timetable below to answer the following questions.

London - Reading - Bristol → Taunton
Exeter - Torbay - Plymouth - Penzance

Mondays to Fridays

London Paddington	**0530**	**0740**	**0815**	**0845**	**0935**	**0945**	**1035**	**1135**	**1235**
Reading	**0610**	**0805**	**0845**	**0913**	**1003**	**1015**	**1103**	**1203**	**1303**
Newbury	—	—	—	—	**1018**	—	—	—	—
Pewsey	—	—	—	—	**1037**	—	—	—	—
Westbury	—	—	—	—	**1056**	—	—	**1248**	**1348**
Castle Cary	—	—	—	—	—	—	—	**1305**	—
Bristol Temple Meads	**0750**	**0915**	**1008**	**1028**	—	**1127**	—	—	—
Taunton	**0843**	**0952**	**1051**	**1108**	**1133**	—	—	**1328**	**1423**
Tiverton Parkway	**0857**	—	—	**1120**	**1145**	—	—	**1340**	—
Exeter St Davids	**0913**	**1016**	**1117**	**1135**	**1200**	**1235**	**1239**	**1355**	**1449**
Dawlish	**0928**	—	—	**1150**	—	—	—	—	—
Teignmouth	**0933**	—	—	**1155**	—	—	—	—	—
Newton Abbot	**0941**	**1040**	**1141**	**1203**	**1224**	—	—	**1419**	**1513**
Totnes	**0952**	**1052**	—	—	**1235**	—	—	**1433**	—
Plymouth	**1023**	**1120**	**1222**	**1240**	**1310**	—	**1339**	**1505**	**1551**
Liskeard	**1051**	**1149**	**1311**	—	—	—	**1404**	**1547**	**1619**
Bodmin Parkway	**1102**	**1201**	**1323**	—	—	—	**1416**	**1559**	**1631**
Par	**1114**	**1211**	**1334**	—	—	—	**1427**	**1611**	**1644**
St Austell	**1121**	**1219**	**1341**	—	—	—	**1435**	**1618**	**1652**
Truro	**1138**	**1237**	**1359**	—	—	—	**1453**	**1636**	**1710**
Redruth	**1150**	**1249**	**1411**	—	—	—	**1505**	**1649**	**1722**
Camborne	**1156**	**1256**	**1417**	—	—	—	**1512**	**1656**	**1729**
St Erth	**1207**	**1307**	**1428**	—	—	—	**1523**	—	**1741**
Penzance	**1217**	**1320**	**1439**	—	—	—	**1535**	**1716**	**1755**

(a) Can he get to Totnes by train to meet his friend Lucy at noon?

(b) What is the earliest time he can arrive in Totnes?

(c) Lucy can meet Farharaz at Newton Abbot. What is the earliest time he can be there?

4. The table gives the distances, in miles, between some towns and cities.

(a) Find the distances between:

(i) Birmingham and Leeds

(ii) Manchester and Bradford

(iii) Sheffield and Coventry.

(b) Nargis travels from Birmingham to Leeds and then to Manchester.

(i) How far does he travel?

(ii) How would the distance he travels be reduced if he went to Manchester and then to Leeds?

	Birmingham	Bradford	Coventry	Derby	Doncaster	Leeds	Manchester
Bradford	108						
Coventry	18	114					
Derby	40	74	40				
Doncaster	93	34	92	54			
Leeds	109	9	110	70	28		
Manchester	80	34	94	59	51	40	
Sheffield	77	37	78	37	18	33	38

(c) Denise is going to Birmingham from Leeds. She must stop in either Bradford or Sheffield on the way. Which is the shorter route?

5. The table gives the distances in km between 3 ports, Calais, St. Malo and Boulogne, and some holiday destinations in Europe.

Calais	St. Malo	Boulogne	Florence	Interlaken	Venice	Quimper	Ile de Re	Biarritz
1422	1560	1430						
811	1169	819	612					
1370	1561	1388	307	563				
729	244	707	1708	1172	1710			
704	344	670	1494	995	1355	400		
1110	729	1152	1358	1233	1504	787	409	

(a) The Eccles family decide to go to Biarritz for their holiday. Which of the three ports (Calais, St. Malo or Boulogne) is closest to Biarritz?

(b) They decide to start their holiday at Calais, and also to visit Venice before they return to Calais. How far do they have to travel in total?

(c) Their friends, the Morse family, decide to travel from St. Malo to Biarritz, then Interlaken and return to Boulogne. How far do they have to travel?

(d) Which holiday destination is closest to St. Malo and which is closest to Calais?

6. In a school 30 students took GCSE exams in both Maths and Physics. Their results are given in this table.

Maths Grade

Physics Grade	**A**	**B**	**C**	**D**	**E**
A	2	3			
B	1	1	4		
C		2	3	2	
D			4	2	2
E		1	0	2	1

(a) How many students got the same grade in both subjects?

(b) How many students got a higher grade in Physics than in Maths?

(c) Which was the most common grade in Physics?

7. The table shows the sports options selected by a group of students in each of their years in secondary school. In each year, each student chose just one sport.

	Hockey	*Football*	*Tennis*	*Swimming*
Year 7	6	18	5	14
Year 8	5	16	7	15
Year 9	7	14	10	12
Year 10	2	12	10	19
Year 11	8	13	12	10

(a) How many chose tennis in Year 8?

(b) How many more chose football in Year 7 than in Year 10?

(c) In which years was football the most popular sport?

(d) (i) In which year was swimming the most popular sport?

(ii) How many students were there in this group?

8. The table below shows the cost of a week at a large holiday centre. The cost varies according to the number of people in the party and the type of accommodation booked. There is a £20 reduction for each child.

		Accommodation Type		
		Saver	*Comfortable*	*Luxury*
Number of people in party	4	£180	£260	£368
	5	£220	£320	£454
	6	£265	£385	£541
	7	£305	£449	£630
	8	£340	£507	£704

(a) How much would it cost for Mr and Mrs Jones and their 4 children to stay in 'Comfortable' accommodation for one week?

(b) How much more would it cost if they booked 'Luxury' accommodation?

(c) How much would they save in 'Saver' accommodation compared with 'Luxury' accommodation?

(d) (i) How much would it cost if two grandparents came with the family and they all stayed in 'Saver' accommodation?

(ii) By how much does this differ from the total in part (a)?

9. In a privatised railway company, there are 84 male conductors and 56 female conductors. Conductors can either be senior or standard. There is a total of 28 senior conductors and there are 48 female standard conductors.

(a) Copy and complete the two-way table to show the number of male and female conductors who are senior or standard.

	Male	***Female***
Standard		
Senior		

(b) Comment on the results.

10. Each student in a class chose *one* sport. The numbers of choices were put in a table.

		Outdoor Sports		**Indoor Sports**	
		Hockey	Tennis	Badminton	Squash
Year 11	Girls	12	10	15	5
	Boys	10	15	7	19
Year 10	Girls	14	9	17	3
	Boys	15	12	11	13

(a) How many students chose hockey?

(b) How many more girls chose tennis than squash?

(c) One girl says that boys usually prefer outdoor sports. Do the figures in the table support this view? Explain your answer.

(SEG)

11. The two-way table shows the number of students achieving grades **A** to **E** in examinations in English and French.

French grade \ **English grade**	E	D	C	B	A
A			1	1	2
B		1		5	2
C		1	7	2	
D		2	3	1	
E	3	1			

(a) How many of the students who achieved grade **B** in English achieved a different grade in French?

(b) How many students achieved the same grade in both subjects.

(c) What does the table suggest about the grades achieved in English and French?

(SEG)

12. St. Margaret's School entered Y11 and Y10 pupils for the NEAB mathematics examination at levels P, Q and R as shown in the table.

(a) How many Y10 pupils have entered for the examination?

(b) (i) What was the total number of pupils entered for level Q?

(ii) What percentage of the pupils entered at level Q were Y10 pupils?

		Level P	Level Q	Level R
Y11	Boys	25	18	7
	Girls	10	45	11
Y10	Boys	0	13	3
	Girls	0	14	1

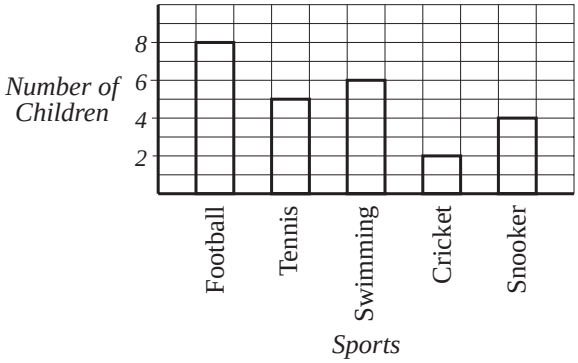
(NEAB)

8.2 Pictograms and Bar Charts

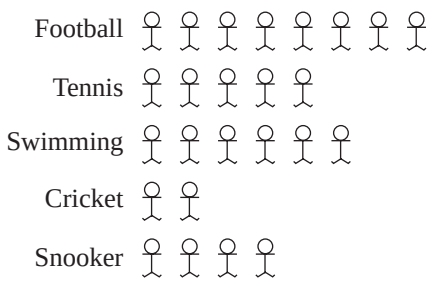
Bar charts and pictograms can be used for displaying data when the data are in discrete categories.

For example, the bar chart and pictogram below show the favourite sports of 25 children.

Bar Chart

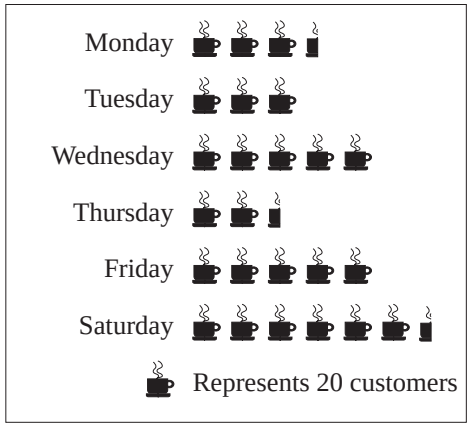


Pictogram



Worked Example 1

The pictogram shows the number of customers using a coffee shop during one week.



(a) How many customers used the shop on Wednesday?

(b) How many customers used the shop on Monday?

(c) How many customers visited the coffee shop during the week?

Solution

(a) For Wednesday there are 5 symbols, so the number of customers was

$$5 \times 20 = 100$$

(b) For Monday there are $3\frac{1}{2}$ symbols, so the number of customers was

$$3\frac{1}{2} \times 20 = 70$$

(c) The total for the week is given by

$$70 + 60 + 100 + 50 + 100 + 130 = 510 \text{ customers.}$$

Worked Example 2

John asked each person in the class what their shoe size was. He obtained these results.

7	5	6	8	4	$5\frac{1}{2}$
$6\frac{1}{2}$	7	8	$7\frac{1}{2}$	$5\frac{1}{2}$	6
$6\frac{1}{2}$	$5\frac{1}{2}$	7	6	$6\frac{1}{2}$	8
7	5	$6\frac{1}{2}$	6	$7\frac{1}{2}$	7
$5\frac{1}{2}$	6	5	$5\frac{1}{2}$	6	$7\frac{1}{2}$

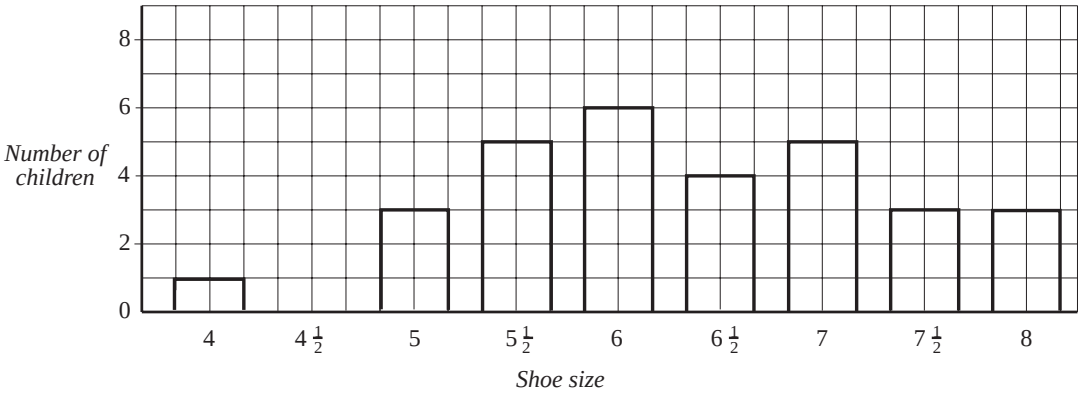
Draw a bar chart to show this data.

Solution

First the data can be entered into a tally chart.

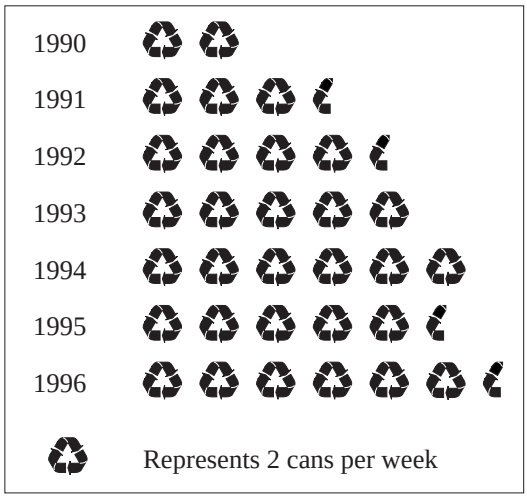
Shoe Size		*Total*
4	|	1
$4\frac{1}{2}$		0
5	|||	3
$5\frac{1}{2}$	𝍸	5
6	𝍸 |	6
$6\frac{1}{2}$	𝍸 |||	4
7	𝍸	5
$7\frac{1}{2}$	|||	3
8	|||	3
		30

The bar chart can be drawn as shown below.



Exercises

1. Jenny kept a record of the average number of cans she recycled each week over a number of years. The pictogram shows her results.



   (a) In which year did she recycle most cans?

   (b) How many cans did she recycle each week in;

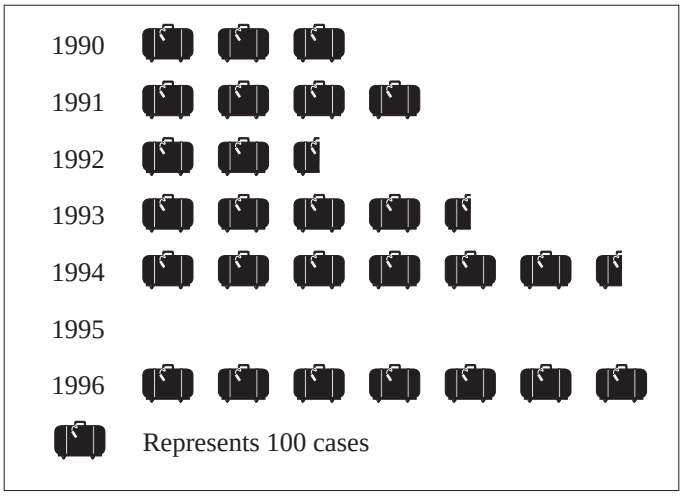
      (i) 1993,

      (ii) 1991,

      (iii) 1994?

   (c) In which year did she recycle an average of 11 cans per week?

2. The pictogram shows how many suitcases were sold by a shop from 1990 to 1996, with one row missing.



   (a) How many cases were sold in 1991?

   (b) What is the smallest number of cases sold in a year?

(c) What is the greatest number of cases sold in a year?

(d) In 1995 a total of 550 cases were sold. How many cases should appear in the missing row?

(e) How many suitcases have been sold altogether?

3. A class conducted a survey to find their favourite ice creams. The results were:

Favourite Ice Cream	*Number of Children*
Solero	9
Magnum	12
Mars	7
Feast	4

(a) Draw a pictogram to show these results.

(b) Represent this information in a bar chart.

What are the advantages of each type of representation of the data?

4. A group of students recorded the number of vehicles passing their school in one hour. The results are recorded below.

Vehicle Type	*Number of Vehicles*
Cars	20
Vans	8
Lorries	3
Motorbikes	5
Buses	2

Represent this information with a bar chart.

5. Draw a bar chart to show the data given in the table about the hours of sunshine per day at a number of resorts.

Resort	*Hours of Sunshine per Day*
Algarve	6
Benidorm	6
Eilat	7
Majorca	5
Mombasa	9
Tenerife	6
Torremolinos	6

6. The children on a school bus were asked which year group they were in. Their replies were:

10	7	7	10	11	9	8	7	8	9
7	9	11	11	8	8	9	7	10	10
11	8	9	7	10	11	11	11	11	7
7	7	8	7	8	9	10	10	9	8

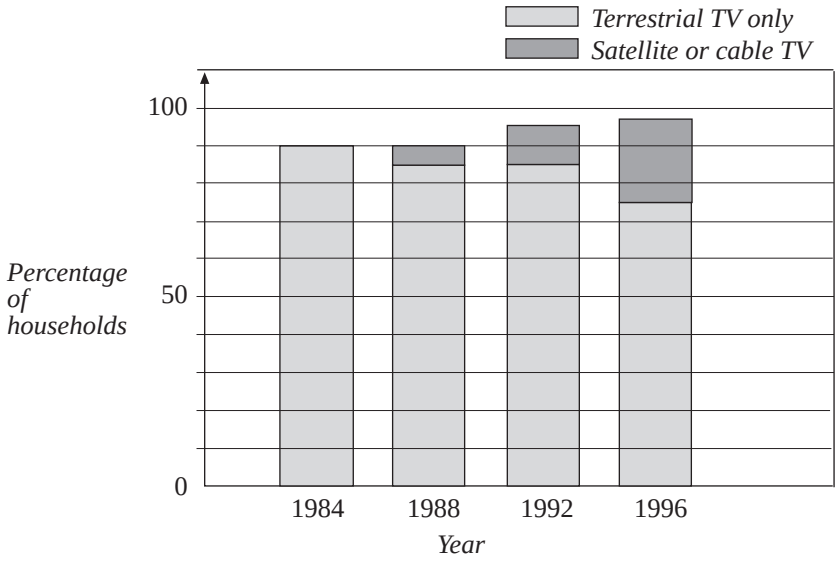
Draw a bar chart to show this data.

7. A headteacher asked a class of Year 7 students how many younger brothers and sisters each student had. The results were:

0	1	2	1	0	0	1	2	1	1
2	0	0	1	1	2	3	4	1	1
2	1	2	0	0	3	2	1	5	1

Draw a bar chart to illustrate this data.

8. A group of households were asked to give information about access to satellite, cable and terrestrial TV. The results are summarised below.



Use this presentation of the results to answer the following questions.

(a) What percentage of households had some form of TV in

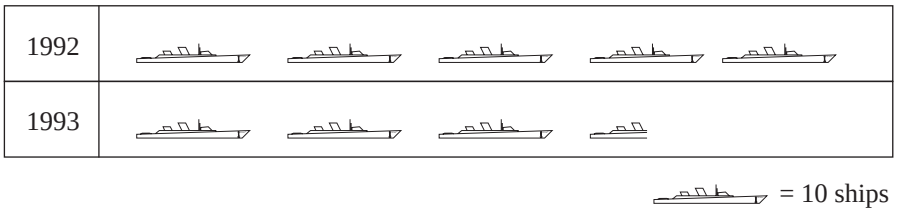
(i) 1984 (ii) 1996?

(b) What percentage of households had satellite or cable TV in

(i) 1984 (ii) 1996?

(c) Comment on the trends in the results from the survey.

9.



The diagram shows how many ships were in Mathsland's navy in 1992 and 1993.

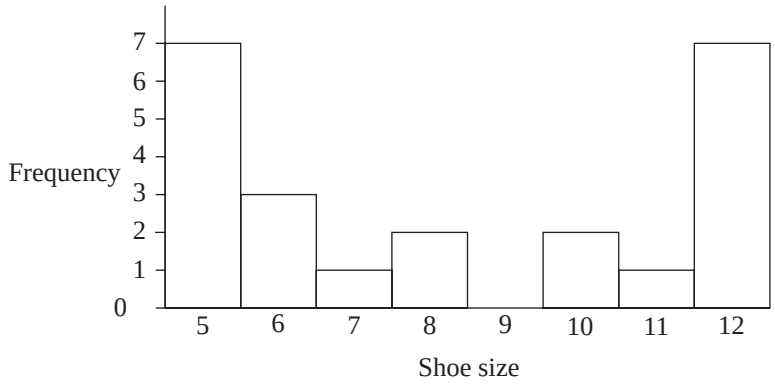
Use the diagram to answer these questions.

(a) How many ships were there in Mathsland's navy in 1992?

(b) How many ships were there in Mathsland's navy in 1993?

(LON)

10. The bar chart below shows the shoe sizes of a group of 16 year old boys.

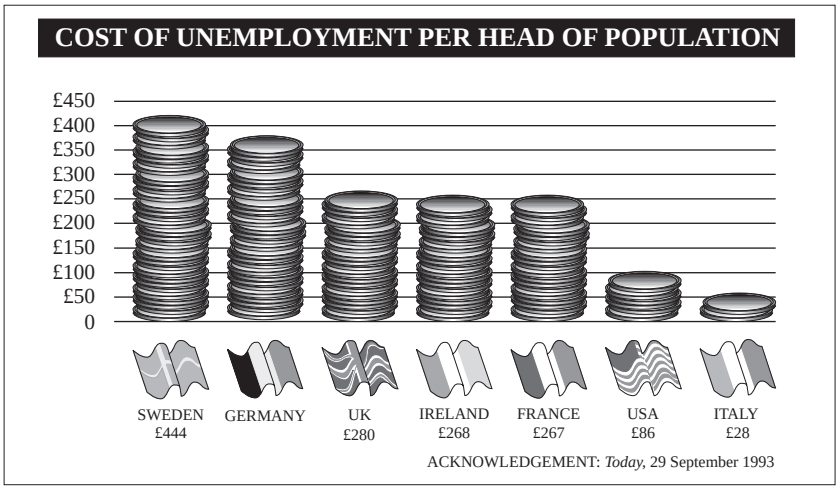


(a) How many boys are there in the group?

(b) Comment on the shape of the bar chart, saying whether or not this is the shape you would expect.

(MEG)

11.



The pictogram represents, approximately, the cost of unemployment per head of population in seven developed countries. The exact figures for six of them are shown under their names.

(a) There are 19 coins in the pile for Ireland? To the nearest whole number, how many pounds does one coin represent?

(b) Estimate the cost of unemployment per head of population in Germany.

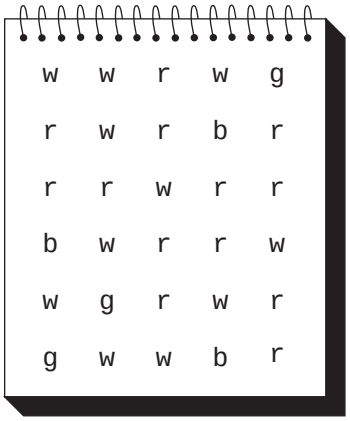
(c) The cost of unemployment per head of population in Japan is £218.

How many coins would there be in the pile for Japan?

(d) Explain why the pictogram is only approximate.

12. Sally did a survey of car colours.

The notebook shows all her results.



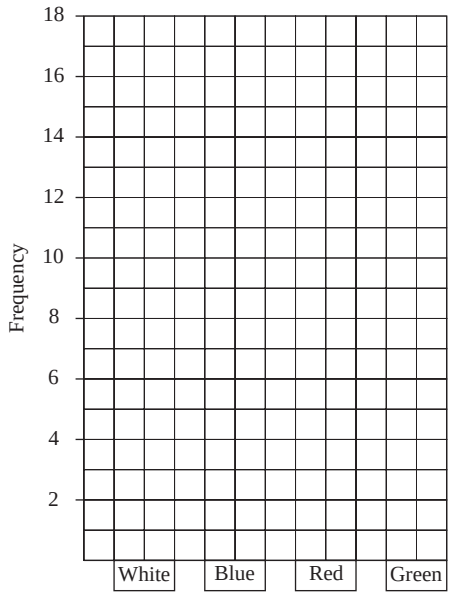
Key:

w white
b blue
r red
g green

(a) Copy and complete the frequency table.

COLOUR	TALLY	FREQUENCY
White		
Blue		
Red		
Green		

(b) Show this information as a bar chart of the form shown here.



(SEG)

8.3 Pie Charts

Pie charts, which represent quantities as sectors of a circle, can be used to illustrate data. They are particularly effective if there is only a small number of items to illustrate. In total a complete circle, i.e. 360°, must always be used.

Worked Example 1

Tracey uses her pocket money of £18 per month in the following way.

Magazines	£4
Sweets	£3
Swimming	£6
Bus fares	£4
Money box	£1

Draw a pie chart to show how Tracey uses her pocket money.

Solution

Tracey has a total of £18.

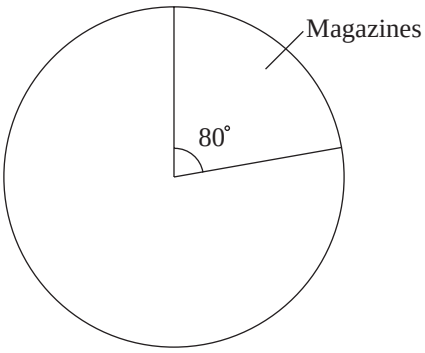
$$\frac{360°}{18} = 20°$$

So $20°$ should be used for each £1. The angles needed are given in this list.

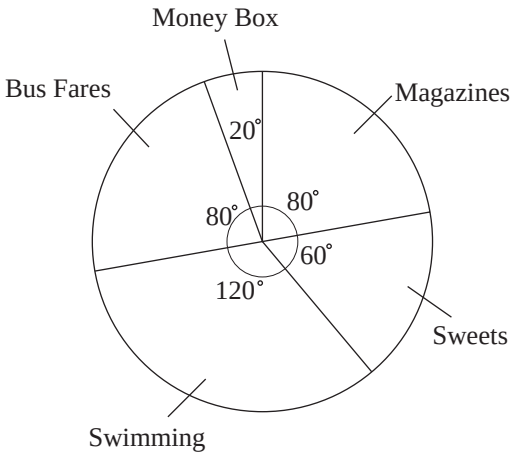
Magazines	$4 \times 20° = 80°$
Sweets	$3 \times 20° = 60°$
Swimming	$6 \times 20° = 120°$
Bus fares	$4 \times 20° = 80°$
Money box	$1 \times 20° = 20°$

The pie chart can now be drawn.

The diagram below shows the first section for magazines.

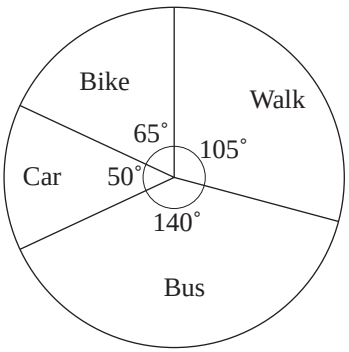


This is the completed pie chart.



Worked Example 2

The pie chart was constructed by asking 72 children how they travel to school.



(a) How many children travel to school by:

(i) car, (ii) bike, (iii) bus?

(b) What percentage walk to school?

Solution

(a) There are 72 children so

$$\frac{360°}{72} = 5° \text{ per child.}$$

(i) The angle for travelling by car is $50°$ so $\frac{50°}{5°} = 10$ children travel by car.

(ii) The angle for travelling by bike is $65°$ so $\frac{65°}{5°} = 13$ children travel by bike.

(iii) The angle for travelling by bus is $140°$ so $\frac{140°}{5°} = 28$ children travel by bus.

(b) The number who walk to school is given by

$$\frac{105°}{5°} = 21$$

so the percentage who walk is

$$\frac{21}{72} \times 100 \approx 29.1\%$$

Note

Alternatively, you could just use the angles in the pie chart to give

$$\frac{105}{360} \times 100 \approx 29.1\%$$

Exercises

1. In an opinion poll 360 people were asked who they would vote for in the next election. Their responses are:

Labour	150
Liberal Democrat	60
Conservative	100
Other	50

Draw a pie chart to show this.

2. Sarah recorded how she spent the last 24 hours. Her results are below.

Sleeping	9 hours
School	7 hours
Homework	2 hours
Watching TV	3 hours
Eating	1 hour
Travelling	2 hours

Draw a pie chart to show this information.

3. Emma has 720 stamps in her stamp collection. She has sorted them into three groups.

UK 400 stamps, Europe 200 stamps, Other countries 120 stamps

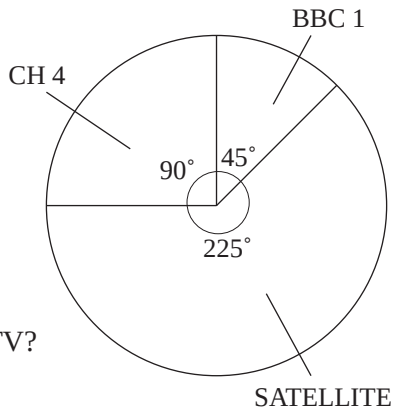
Draw a pie chart to show this information.

4. The 30 pupils in a class state their favourite sport. Their results are listed below.

Snooker	3
Football	9
Netball	6
Squash	2
Tennis	10

Draw a pie chart to show this information.

5. The pie chart shows how the time Ron spends watching television is split between different channels, for one day.

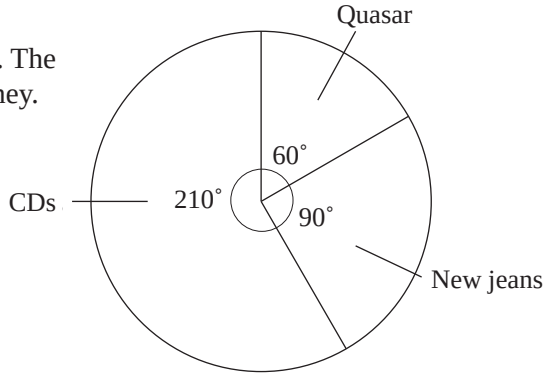


Ron spends 1 hour watching BBC1.

(a) How long does he spend watching CH4?

(b) How long does he spend watching satellite TV?

6. Ahmed was given £60 on his birthday. The pie chart shows how he spent this money.



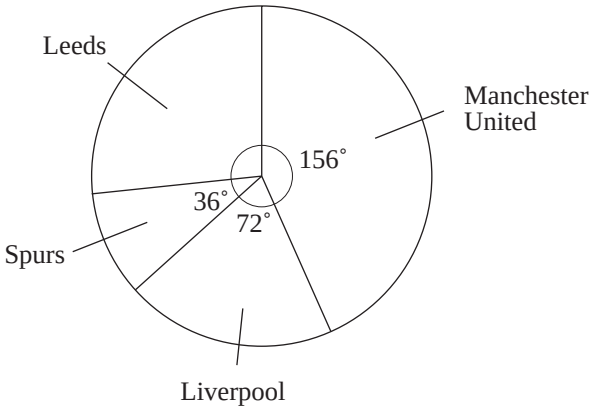
How much did he spend on:

(a) Quasar,

(b) his new jeans,

(c) CDs?

7. The pie chart shows the football teams supported by a class of children. There are 3 children who support Spurs.



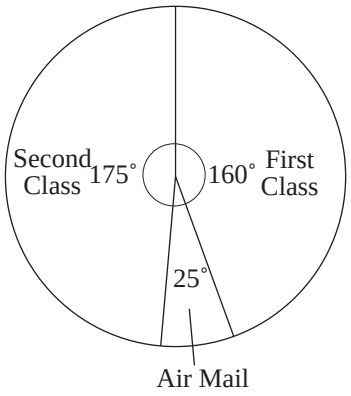
(a) What is the angle representing Leeds?

(b) How many children support Liverpool?

(c) How many children are there in the class?

(d) How many children support Manchester United?

8. A postman collects 720 letters from the letter boxes in a small town. They are sorted into First Class, Second Class and Air Mail. The pie chart shows the different numbers of each type. How many letters of each type did he collect?



9. Sita spent £90.
The table shows what she spent it on.

Items	Amount spent
Bus fares	£12
Going out	£25
Clothes	£30
Records	£15
Others	£8
Total Spending	£90

Sita is asked to construct a pie chart to show her spending.

(a) Work out the angle of each sector in the pie chart.

Items	Angle of sector
Bus fares Going out Clothes Records Others	
Total of angles	360°

(b) Construct the pie chart to illustrate the data.

(c) What fraction of Sita's spending was on clothes?

(LON)

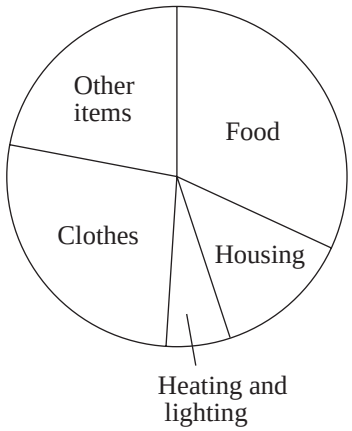
10. Arthur spends £180 per week.

Item	Spending £
Food	30
Heating and lighting	12
Clothes	20
Other items	20
Housing	98
Total spending	180

The way in which he spends his money is shown in the table.

(a) Draw a pie chart to show how Arthur spends his money.

This pie chart shows how the average person spends money.



(b) Describe **one** way in which Arthur's spending differs from the average person's spending.

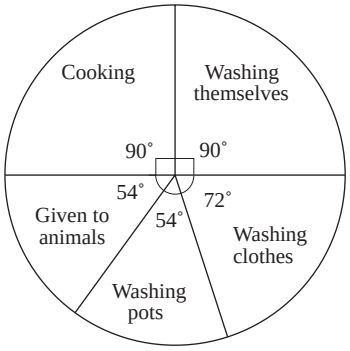
(SEG)

11. This question is about the way water is used in two Mozambique villages.

(a) In village *A*, 324 litres of water are used each day.

The pie chart shows how the water is used.

USE OF WATER IN VILLAGE *A*



(i) How much water (in litres) is used each day for cooking?

(ii) What fraction of the water used is given to animals?

(b) In village *B*, the water is used as follows:

Cooking	20%
Washing themselves	50%
Washing clothes	20%
Washing pots	10%

Represent this information in a pie chart.

(MEG)

12. (a) (i) 500 000 cars were stolen in England and Wales in 1990.

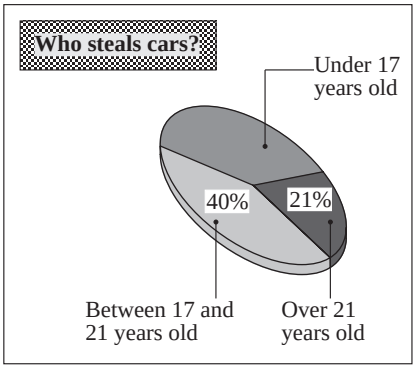
Write the number 500 000 in words.

(ii) 40% of the cars stolen in 1990 were never found.

Calculate the number of cars that were never found.

(b) The chart shows information about the people who stole cars in 1990.

Use the information in the diagram to answer these questions about car theft in 1990.



(i) What percentage of the cars stolen were taken by people aged under 17?

(ii) Which age group was the least likely to be involved in stealing cars?

(iii) What is the probability that a stolen car was taken by a person who was aged 17 or over?

Give your answer as a decimal or a fraction.

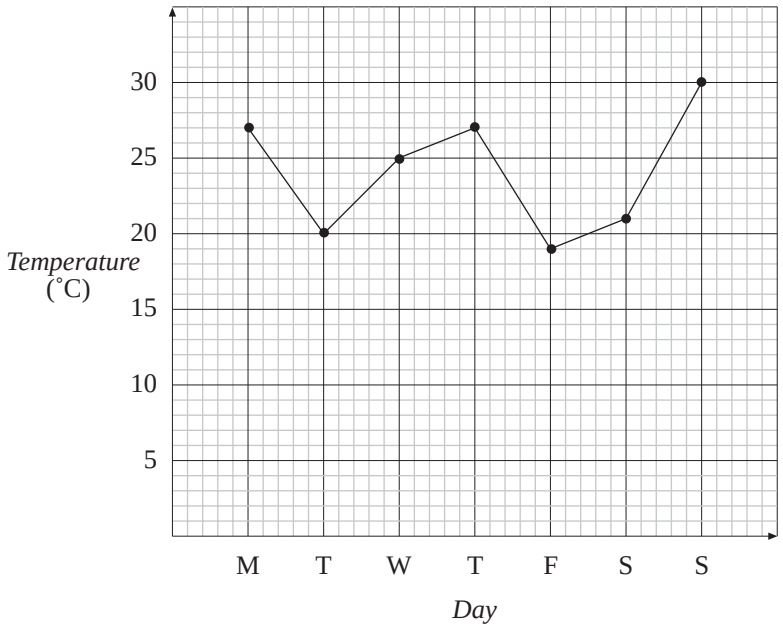
(NEAB)

8.4 Line Graphs

A *line graph* is drawn by plotting data points and joining them with *straight* lines. It is really only the actual data points that count, but by drawing the lines you get a better impression of the trend in the data points. This method of representation is particularly useful when illustrating trends over time.

Worked Example 1

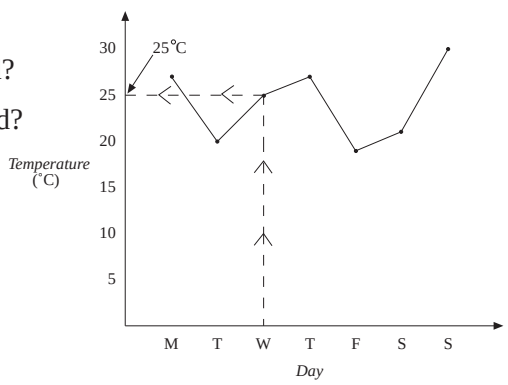
Stuart recorded the temperature in his greenhouse at 6 pm each day for a week. His records are shown on this line graph.



(a) What was the temperature on Wednesday?

(b) What was the lowest temperature recorded?

(c) What was the highest temperature recorded?

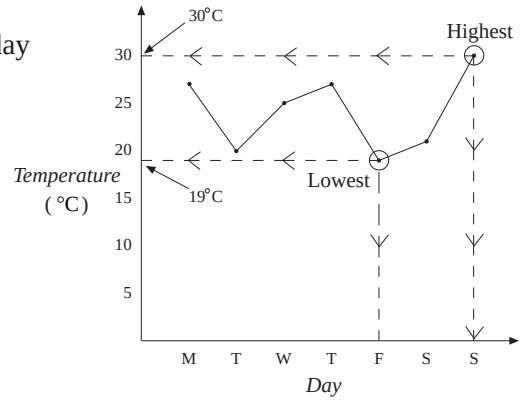


Solution

(a) For Wednesday the temperature can be read as $25°C$.

(b) The lowest temperature occurred on Friday and was 19°C.

(c) The highest temperature occurred on Sunday and was 30°C.



Worked Example 2

As part of a science project Evan records the height of a plant every week. His results are in this table.

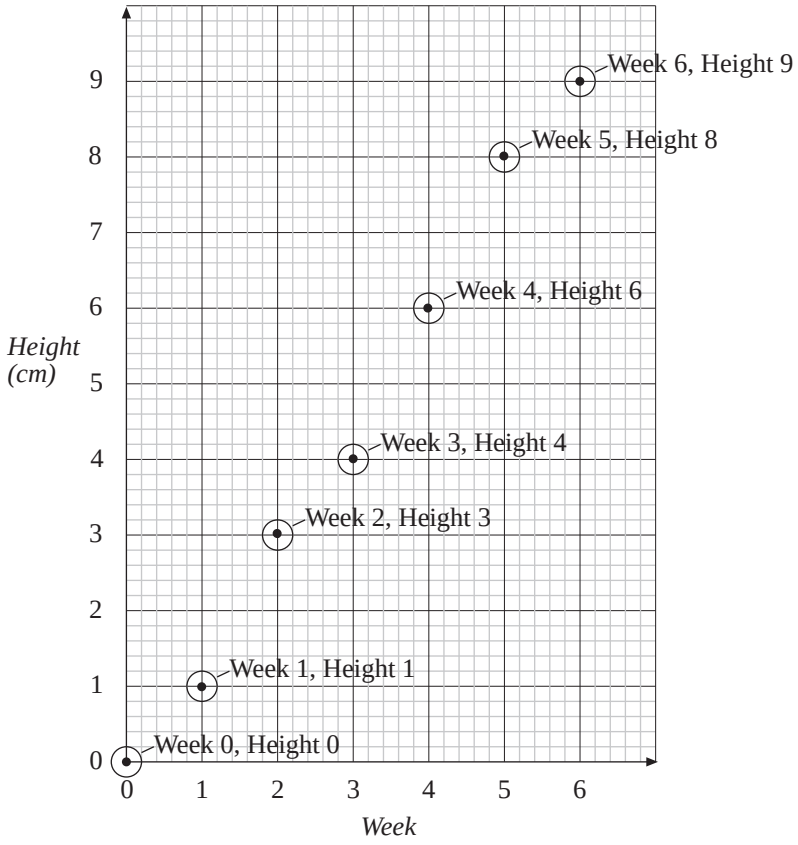
Week	0	1	2	3	4	5	6
Height (cm)	0	1	3	4	6	8	9

Draw a line graph.

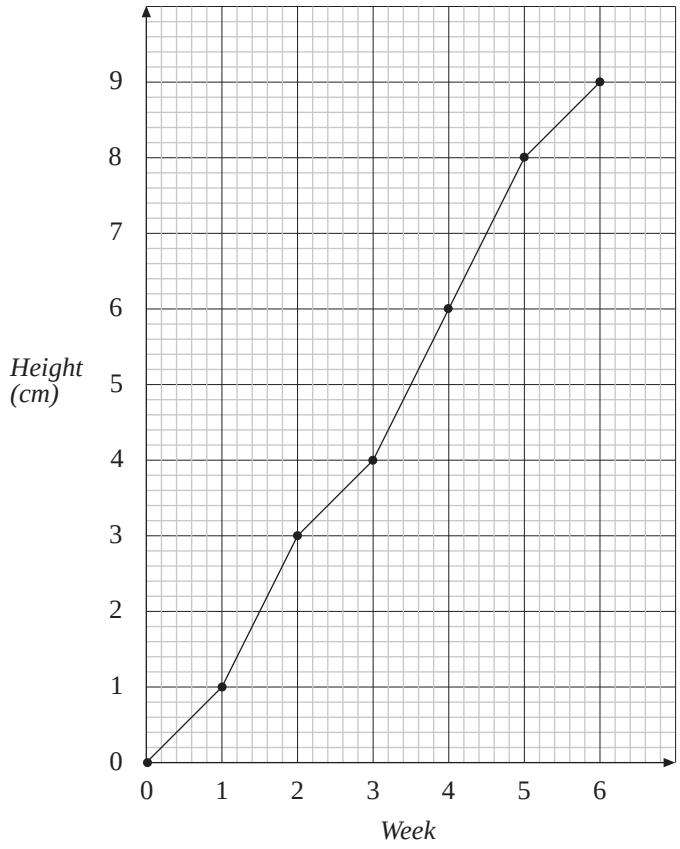
Solution

First draw a suitable set of axes.

Then plot a point for each measurement as shown below.

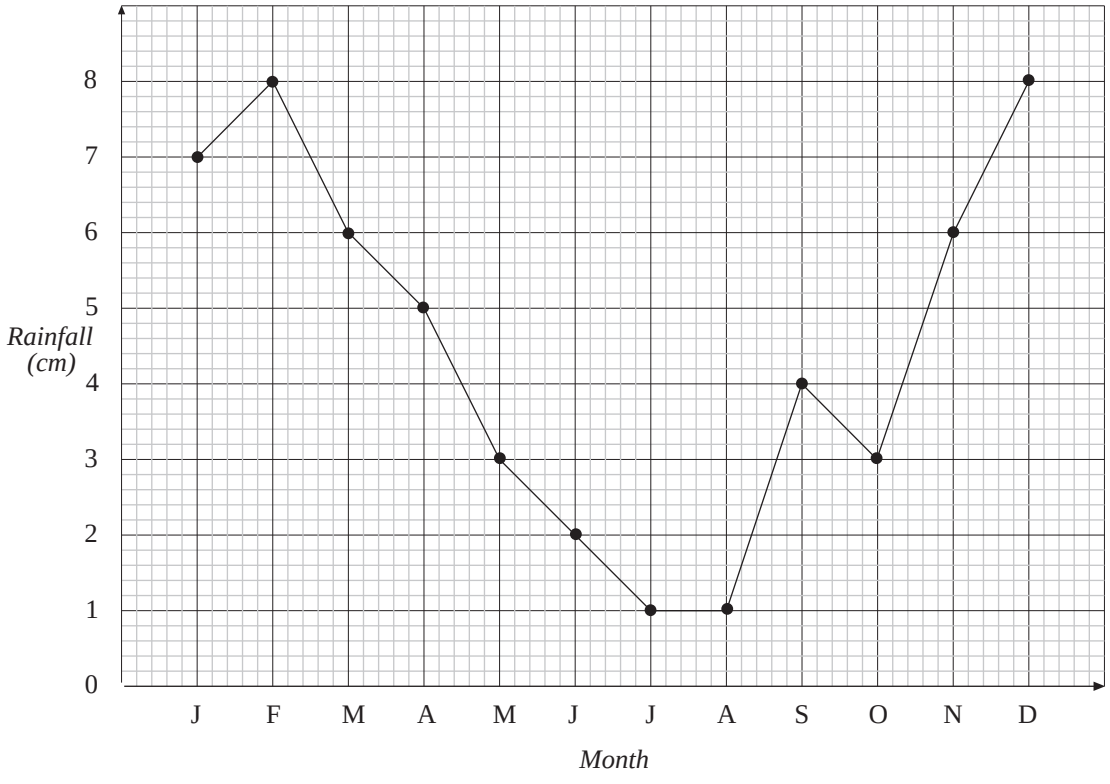


The points can then be joined with straight lines as shown in the next graph.



Exercises

1. The line graph shows the monthly rainfall for a town.



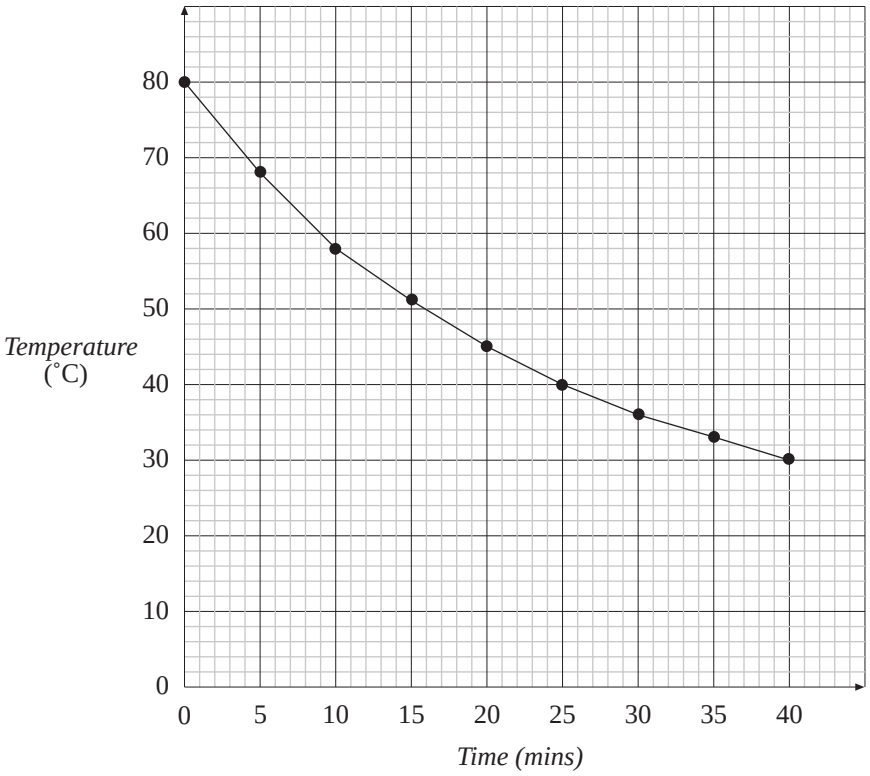
(a) How much rain was there in September?

(b) In which month was the rainfall 5 cm?

(c) Which months were the wettest?

(d) Which months were the driest?

2. A mug was filled with hot water and the temperature was recorded every 5 minutes. The graph below shows the results.



(a) What was the temperature after 25 minutes?

(b) What was the temperature at the start of the experiment?

(c) When was the temperature $45°C$?

(d) How long did it take for the temperature to drop from $68°C$ to $36°C$?

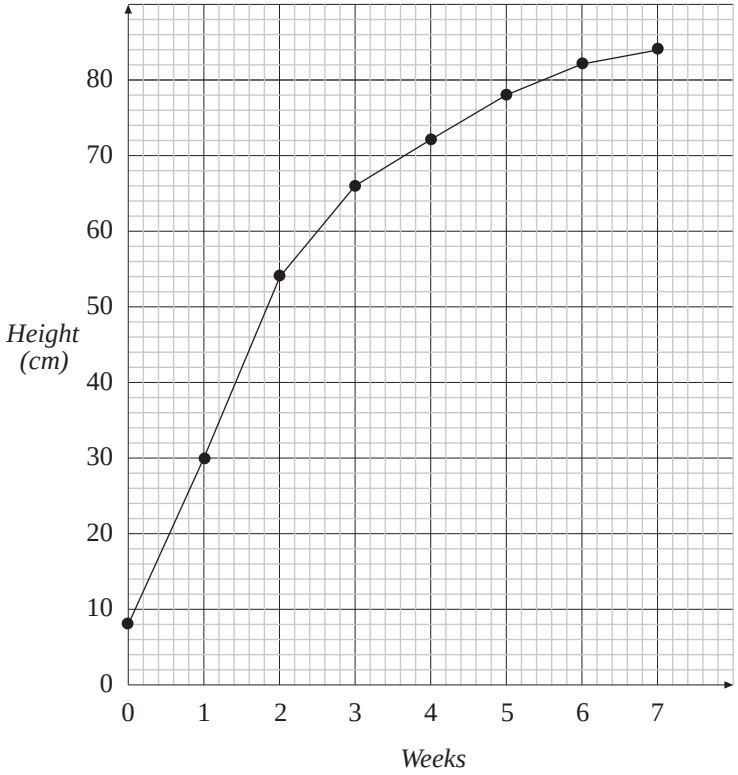
3. The following graph shows how the height of a sunflower plant changed since it was planted in a garden.

(a) What was the height of the plant when it was planted in the garden?

(b) How much did the plant grow in the first week?

(c) What is the greatest height that the graph shows?

(d) How long did it take for the height to increase from 54 cm to 78 cm?



4. Paul recorded the temperature outside his house at 8.00 am every day. His results are in the table.

Day	M	T	W	T	F	S	S
Temperature (°C)	8	5	4	6	7	5	3

Draw a line graph for this data.

5. Karen counted the number of cars that drove past her while she was waiting at the bus stop each morning on her way to work.

Day	M	T	W	T	F	S
Number of cars	18	12	22	36	4	10

Draw a line graph for this data.

6. Anna recorded the time it took her to walk to school every day for a week.

Day	M	T	W	T	F
Time taken (mins)	8	9	15	12	7

Draw a line graph for this data.

7. Stuart is training to run a marathon. Each week he recorded the time it took him to run 5 miles.

Week	0	1	2	3	4	5	6
Time (mins)	52	50	46	44	40	36	34

Draw a line graph for this data.

8.5 Questionnaires and Surveys

Note

When designing a questionnaire to use as part of a survey, bear in mind the following guidelines.

- The questionnaire should try to find out the information you need.
- You should know how you are going to collect your responses.
- Questions should be clear and concise with no ambiguity.
- Do not ask for information you do not need.
- Allow for all possible responses.
- Questionnaires should be fair and not biased in any way.
- The people asked to complete the questionnaire should be from a variety of backgrounds.

It should also be noted that if you need to design a questionnaire for your project work, it is strongly recommended that you first try it out with a small number of people. This is called a *pilot* survey.

Questions can be designed in two distinct ways, namely those that require a specified response to be chosen from a number of options or by giving a number, and those that allow more detailed responses. The first group are often referred to as *closed* questions; the second as *open* questions. Here are some examples.

Closed

"Did you watch the football match on TV last night?" YES / NO

"How many hours of TV did you watch last night?"

Choose from '0 – 1, 1 – 2, 3 – 4, 4 – 5, more than 5'

The second example of a closed question shows an example with multiple responses – by which we mean more than 2! So 'YES / NO' is not regarded as a multiple response.

There is though a problem with these responses as there is potential overlap. i.e. in which category do you place 1 hour or 2 hours, etc.? It should be made clear, i.e. 0 – 1 includes everything up to and including 1, etc.

Open

"What sort of TV programmes do you like to watch, and why?"

"Where did you go for your holiday last year?"

The second example is *good f*or a *face to face* discussion, as it allows a wide range of answers. The questioner will then be able to react to these answers, and continue the discussion. This example is *bad* for a *questionnaire* for the same reason! Any answer is possible but the writer of the questionnaire does not have the chance to follow up answers. It could though be modified and become a closed question with multiple responses: for example,

"Please tick a box to show where you went on holiday last year."

☐ *UK* ☐ *USA* ☐ *France* ☐ *Italy*

☐ *Greece* ☐ *Other (please say where)* _____

Worked Example 1

Consider this questionnaire:

Are you concerned about the environment?

Are you concerned about the level of pollution caused by cars?

Do you think the health of young children is at risk due to exhaust fumes from cars?

Is there too much congestion in the city centre?

Is public transport under-used?

Do you think cars should be banned from the city centre?

Comment on the questions given here.

Solution

This questionnaire is very biased and has been designed to lead people towards answering 'yes' to the last question. There are no questions about the advantages of cars, the problems of using public transport or other ways of reducing pollution.

Worked Example 2

What is wrong with the following survey, used to find the favourite sports of the students in one school?

Which is your favourite sport?

Tennis ☐
Rugby ☐
Netball ☐
Basketball ☐
Swimming ☐
Snooker ☐
Other ☐

Solution

The question used in this survey does not cover all the possible responses well. Many students might have, for example, football as their favourite sport and so would tick the box marked 'Other'. This would give unhelpful results. Also, students are asked to make only one choice although they may well like more than one sport.

The following wording would produce more useful responses.

"Please tick one box in each row to show your preference."

	I really enjoy this sport	*I like this sport*	*I've no great preference*	*I dislike this sport*	*I hate this sport*
Tennis					
Rugby					
Netball					
Basketball					
Swimming					
Snooker					
Please put any other sport that you like here					

Exercises

1. Consider the following two sets of questions.

Is meat a good source of protein?	*Is all meat obtained from dead animals?*
Is protein an important part of your diet?	*Do you like killing animals?*
Do you like eating meat?	*Do you like eating meat?*

(a) Comment on the way that the last question might be answered in each case.

(b) Write a better set of questions to find out if people like eating meat.

2. Design a questionnaire that would encourage students at a school to say:

   (a) that school uniform is a good policy,

   (b) that school uniform should be abandoned.

3. Wendy asks her class the following question.

> *Which of these breakfast cereals do you like best?*
>
> Cornflakes ☐ Porridge ☐ Bran Flakes ☐

   She then says that her class's favourite breakfast cereal is Cornflakes.

   (a) Is her conclusion valid?

   (b) Criticise her question.

   (c) Write a better question for her to use.

4. Design a questionnaire which you could use to find out:

   (a) if people think they have a good public transport service,

   (b) whether the music tastes of girls and boys are different,

   (c) whether younger people are more likely than older people to be vegetarian,

   (d) who might win the next General Election.

   Use your questionnaire to collect some data and present your results.

5. You have been asked to estimate what percentage of all cars on the road have each registration letter (M, N, etc.).

   (a) Describe how you would collect the data for a survey to answer this problem.

   (b) Describe any problems that might arise as you collect data.

6. Akiko did a survey to find out which colours of cars are popular.

   She found that red was the most popular colour. She did not find any cars that were purple or pink.

   Think about a survey which ***you*** could do. It must not be about the colour of cars.

   (a) What is your survey about?

   (b) Write down two things which you might expect to find out.

(MEG)

7. In a survey of community life on a new housing estate the following question is suggested.

   (A) "What do you most like about living here?"

   An alternative is proposed.

(B) "Tick the box which describes why you most like living here."

Design of houses ☐
Friendliness of neighbours ☐
More open space ☐

Give one advantage of each form of question.

(SEG)

8. A survey is done to find out in which sports pupils take part.

The results of the survey are shown below.

	% Boys	% Girls
Athletics	**37.1**	**33.3**
Basketball	**12.7**	**4.5**
Cycling	**71.7**	**60.5**
Dancing	**4.9**	**31.5**
Football	**72.0**	**14.5**
Gymnastics	**5.0**	**16.3**
Hockey	**6.5**	**3.5**
Judo	**16.0**	**6.7**
Netball	**3.3**	**35.0**
Roller-skating	**7.3**	**26.0**
Rugby	**11.8**	**2.5**
Swimming	**53.1**	**55.0**

(a) Which two sports are the most popular with **both** boys **and** girls?

(b) The question asked in the survey was:

Which sports do you take part in?

You want to find out more about pupils' involvement in sport.

Write down another question that you could ask.

(NEAB)

9. The school governors are worried about road safety outside the school gates.

They think there ought to be a Pelican Crossing.

To decide on this, they need to find out how much traffic comes along the road at different times of day and on different days of the school week.

Design an observation sheet to gather this information.

(NEAB)

10. Mee Ling thinks that pupils who come to school by bus are more likely to be late than those who do not travel by bus.

In order to test whether or not this is true, she carries out a survey on 100 pupils, from years 7 and 8, for 5 consecutive Tuesdays.

The results are shown in the following table.

METHOD OF TRAVEL	NUMBER OF STUDENT-DAYS	NUMBER OF LATES
Bus	150	40
Cycle	50	10
Car	100	22
Walk	200	25
TOTAL	500	97

(a) Do the results show that Mee Ling is correct?
Show the working on which you base your answer.

(b) Suggest 3 ways in which Mee Ling could have improved her survey.

(c) A pupil is selected at random from Year 7.

Mee Ling stated:

"The table shows that the probability that this pupil walks to school on Tuesdays is $\frac{2}{5}$."

Would you describe this statement as

correct,
or about right,
or wrong?

Choose one of the three alternatives and give reasons for your choice.

(MEG)

11. (a) A headline in a newspaper this year stated:

Students skip Breakfast

Our survey shows that few students are eating cereals, fruit, or bread for breakfast.

In fact they are eating nothing at all!

You are asked to conduct a survey to find out what students eat for breakfast.

Design an observation sheet to collect the data you need.

Invent the first 20 entries on your data sheet.

(b) The newspaper made the following statement about the eating habits of teenagers.

Only one in a hundred teenagers eat fruit and vegetables each day. Over half eat no vegetables other than chips.

You are asked to find out whether this statement is true in your area.

Give three questions you could ask teenagers to see if what the article says is true in your area. *(NEAB)*

12. A bus company attempted to estimate the number of people who travel on local buses in a certain town. They telephoned 100 people in the town one evening and asked 'Have you travelled by bus in the last week?'

    Nineteen people said 'Yes'. The bus company concluded that 19% of the town's population travel on local buses.

    Give 3 criticisms of this method of estimation. *(MEG)*

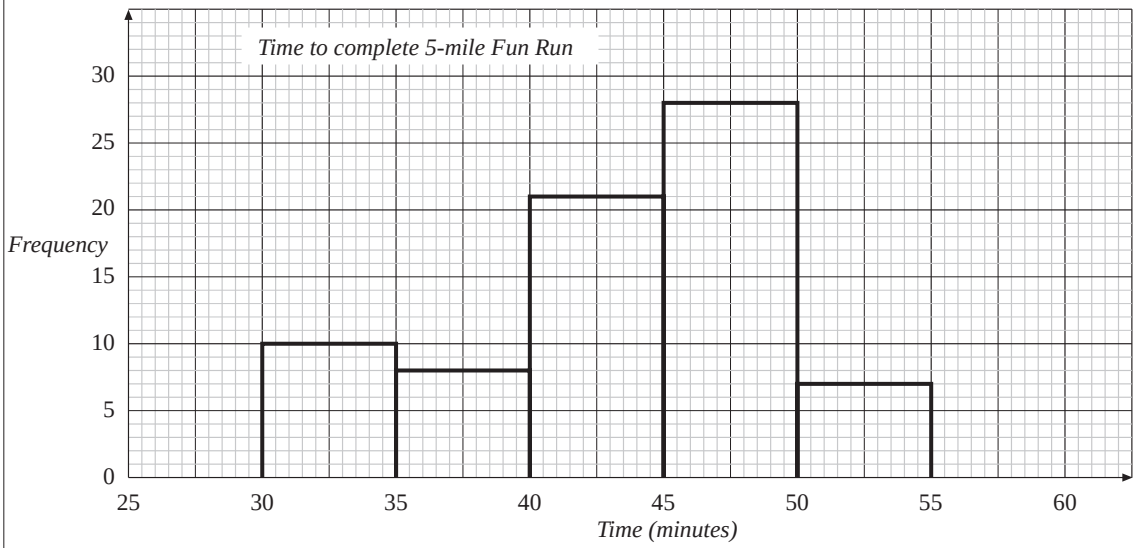
Just for Fun

A man buys a brand new watch. He finds that it is 30 seconds faster per hour than his old grandfather clock. However, the grandfather clock is slower by 30 seconds per hour than the official standard time. Is the new watch accurate?

The man decides to adjust the time of the new watch and the grandfather clock so that it coincides with the official time at 6 am one morning. Give the time shown by the watch and the grandfather clock at 1400 that day.

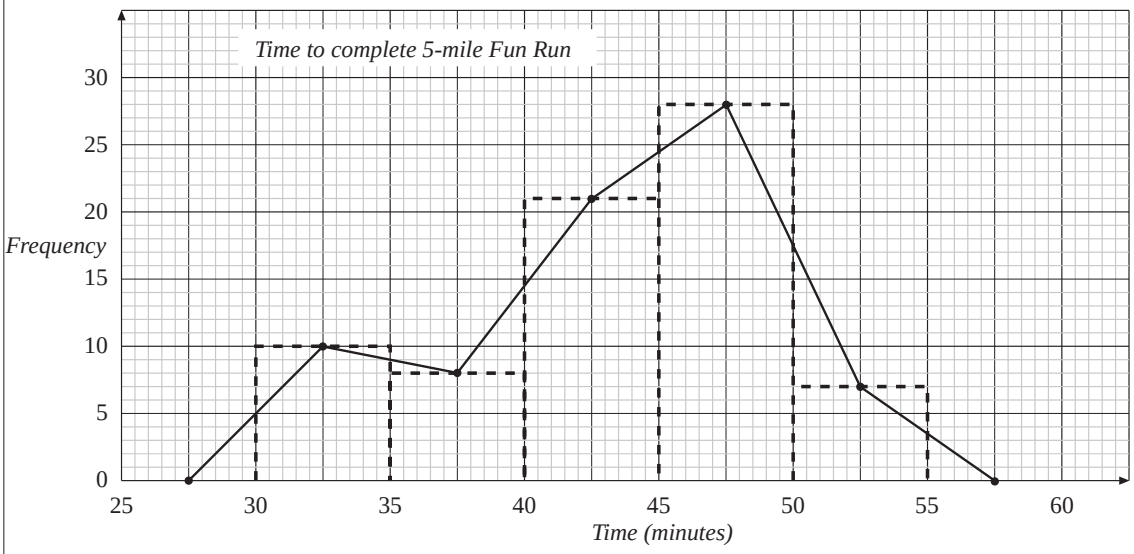
8.6 Frequency Graphs

For *continuous data*, when any value over a range of values is possible, a *frequency graph* like the one below should be used, rather than a bar chart which is used for discrete data.



A graph like this is often called a *histogram*, and is characterised by having a continuous scale along the horizontal axis. Note that in this case the widths of the bars are all the same, but this is not always the case, as you will see in the next section. Care though must be taken about the end points. For example, the first *class interval* (in minutes) would normally be $30 \leq \text{time} < 35$, so that a time of 35 minutes would be in the second class interval.

A *frequency polygon* could also be used to show the same data, as on the following graph. Note how it is related to the histogram.



Worked Example 1

Use the data shown on the graphs above to answer these questions.

(a) How many people completed the Fun Run in between 40 and 45 minutes?

(b) How many people completed the Fun Run in less than 40 minutes?

(c) How many people completed the Fun Run in less than 1 hour?

Solution

(a) The 40-45 minute interval contains 21 people.

(b) The 30-35 and 35-40 minute intervals must be considered.
There are 10 people in the 30-35 minute interval.
There are 8 people in the 35-40 minute interval.
So there are $10 + 8 = 18$ people who complete the run in less than 40 minutes.

(c) The number in each interval is needed.
So the number of people is:

$$10 + 8 + 21 + 28 + 7 = 74$$

Worked Example 2

A group of students measured the reaction times of 50 other students. The times are given below correct to nearest hundredth of a second.

0.44	0.32	0.31	0.47	0.27	0.31	0.40	0.28	0.16	0.26
0.33	0.46	0.41	0.33	0.31	0.28	0.38	0.29	0.17	0.26
0.29	0.40	0.29	0.24	0.41	0.22	0.25	0.47	0.31	0.36
0.49	0.21	0.42	0.43	0.28	0.36	0.24	0.37	0.34	0.27
0.49	0.16	0.29	0.30	0.41	0.27	0.29	0.28	0.40	0.42

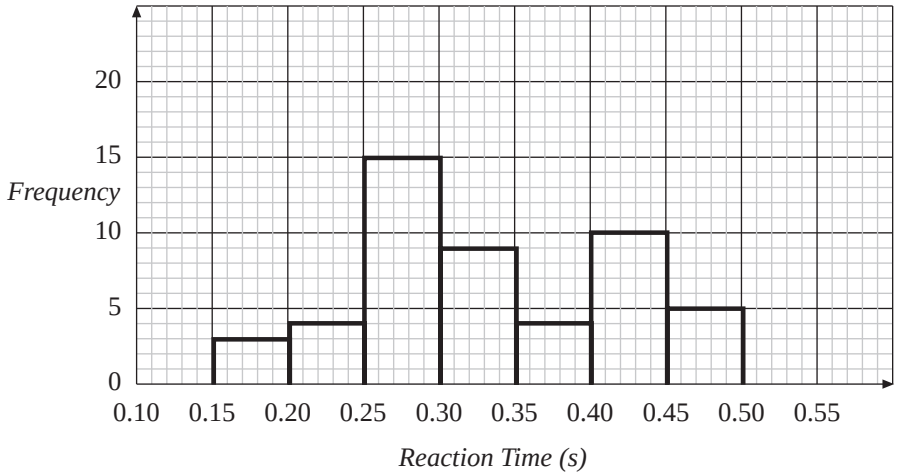
Draw a histogram for this data.

Solution

First the data must be collected into groups, using a tally chart.

Reaction Time(s)	*Tally*	*Frequency*
$0.15 \le t < 0.20$	𝍸	3
$0.20 \le t < 0.25$	𝍸	4
$0.25 \le t < 0.30$	𝍸 𝍸 𝍸	15
$0.30 \le t < 0.35$	𝍸 𝍸	9
$0.35 \le t < 0.40$	𝍸	4
$0.40 \le t < 0.45$	𝍸 𝍸	10
$0.45 \le t < 0.50$	𝍸	5

Now that the data has been collected in this way, the histogram below can be drawn.



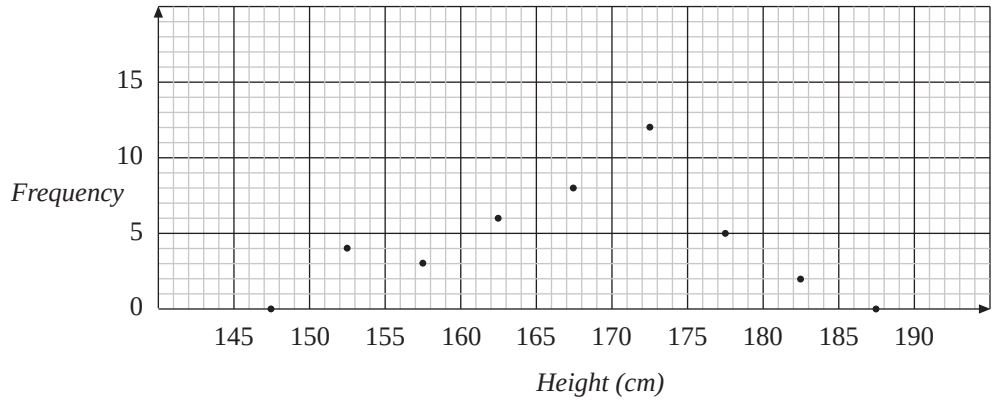
Worked Example 3

Draw a frequency polygon for the data on the height of children given in the table below.

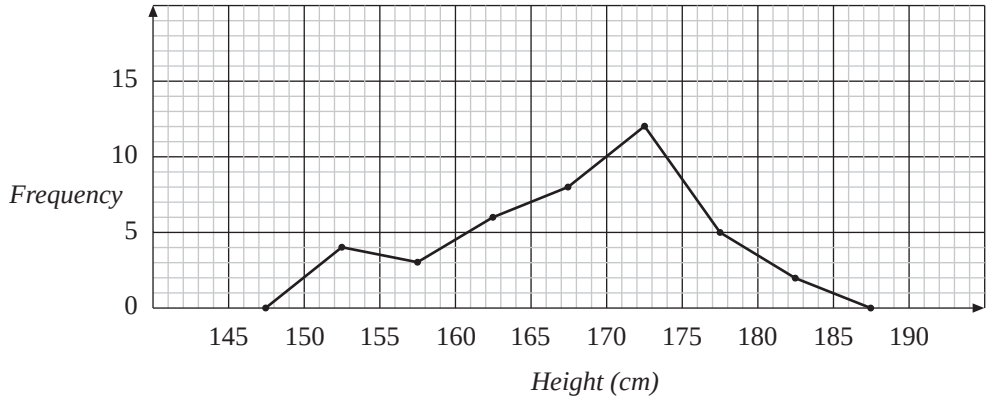
Height	*Frequency*
$150 \le h < 155$	4
$155 \le h < 160$	3
$160 \le h < 165$	6
$165 \le h < 170$	8
$170 \le h < 175$	12
$175 \le h < 180$	5
$180 \le h < 185$	2

Solution

Points should be placed above the centre of each interval. The height is given by the frequency. The following graph shows these points.

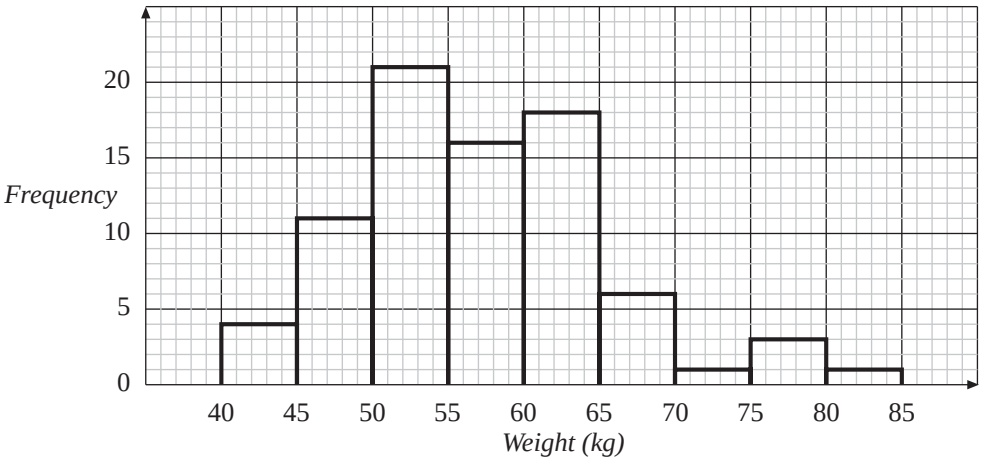


Note that points have been placed on the horizontal axis in the intervals that have frequencies of 0. The points can then be joined to give the frequency polygon below.



Exercises

1. The histogram below shows how the weights of children in one year group were distributed.



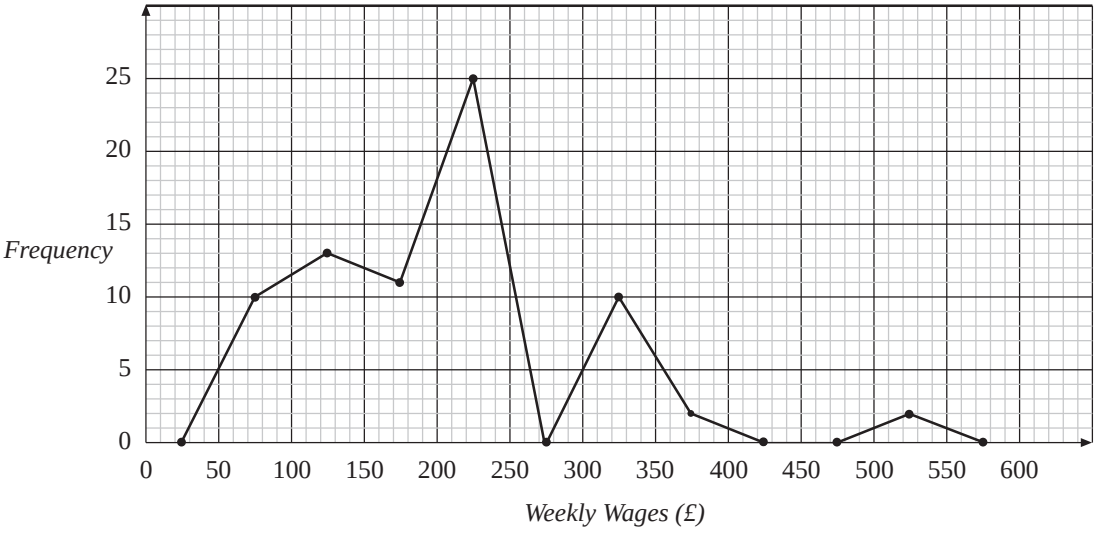
(a) How many children had a weight greater than 70 kg?

(b) How many children had a weight between 50 and 65 kg?

(c) How many children had a weight less than 50 kg?

(d) How many children were there in the year group?

2. The frequency polygon shows the weekly wages of a large firm.



(a) How many people earn between £300 and £350 per week?

(b) How many people earn between £100 and £300 per week?

(c) How many people are employed by the firm?

(d) What are the largest and smallest possible weekly wages that the graph shows could be paid?

3. An orchard contains 100 apple trees. The weight of apples produced by each tree in one year was recorded. The results are given in the table .

Mass of apples (kg)	*Frequency*
$50 < m \leq 60$	5
$60 < m \leq 70$	7
$70 < m \leq 80$	13
$80 < m \leq 90$	10
$90 < m \leq 100$	20
$100 < m \leq 110$	22
$110 < m \leq 120$	18
$120 < m \leq 130$	5

Draw a histogram for the data.

4. A psychologist uses a test in which people have to solve a puzzle. He records the time it took people to solve the puzzle.

Time taken (mins)	$0 \leq t < 1$	$1 \leq t < 2$	$2 \leq t < 3$	$3 \leq t < 4$	$4 \leq t < 5$
Frequency	5	32	18	7	12

Draw a histogram for the data.

5. The finishing times for a cross country race were recorded to the nearest minute.

Draw a suitable histogram for the data.

23	38	43	47
27	39	43	48
31	39	43	48
32	40	43	48
32	40	44	50
32	40	44	50
33	41	46	51
34	41	46	51
35	42	46	52
37	42	47	53

6. At the end of a football season a newspaper calculated the average number of goals scored per match for 100 top footballers.

2.7	1.4	2.9	1.3	2.1	2.1	2.9	2.1	1.7	3.1
1.2	2.0	0.6	2.2	2.2	2.2	2.7	1.6	1.3	1.9
1.3	1.4	1.9	2.1	1.8	0.9	2.1	2.3	1.9	1.3
1.3	0.3	1.2	2.0	1.4	1.7	2.4	1.9	1.7	1.5
2.7	2.2	0.7	1.5	1.6	1.4	2.7	2.0	1.9	2.7
2.1	1.6	0.7	1.5	2.0	0.9	1.6	1.6	1.5	2.6
1.1	2.2	1.8	1.5	1.8	2.6	0.2	1.2	1.4	1.9
1.7	1.4	2.1	2.2	1.5	2.1	2.4	0.5	0.9	1.4
1.8	0.6	1.9	2.6	1.6	2.1	2.0	1.8	1.3	2.0
1.2	2.2	1.4	2.1	1.2	0.4	2.6	1.9	0.9	2.1

Use the data given to draw a suitable histogram and then draw a frequency polygon on top of the histogram.

7. Two students recorded the time it took drivers of cars to find a space and park in a car park. They also noted if the drivers were male or female.

Time Taken (mins)	$0 < t \leq 2$	$2 < t \leq 4$	$4 < t \leq 6$	$6 < t \leq 8$	$8 < t \leq 10$	$10 < t \leq 12$
Male Drivers	4	11	24	11	3	2
Female Drivers	0	16	21	5	5	8

(a) On the same set of axes draw frequency polygons for male and female drivers.

(b) What evidence does the graph provide to support the claim that females take longer to find a space and park than males?

8. The age distribution in a town is given in the table.
Draw a histogram to show the data.

Age	*Frequency*
$0 \le a < 10$	1800
$10 \le a < 20$	1500
$20 \le a < 30$	1450
$30 \le a < 40$	1600
$40 \le a < 50$	1250
$50 \le a < 60$	1150
$60 \le a < 70$	800
$70 \le a < 80$	500
$80 \le a < 90$	150

9. A hire company owns three types of car which are classified as small, medium and large. The distance travelled by each car is always recorded.

Distance (miles)	$0 < t \le 100$	$100 < t \le 200$	$200 < t \le 300$	$300 < t \le 400$	$400 < t \le 500$
Small Cars	80	50	30	2	1
Medium Cars	30	45	67	70	10
Large Cars	5	12	16	24	12

(a) On the same set of axes draw frequency polygons for each type of car.

(b) Comment on the graphs you have drawn.

10. A large number of children entered a mathematics competition. Their scores are listed below.

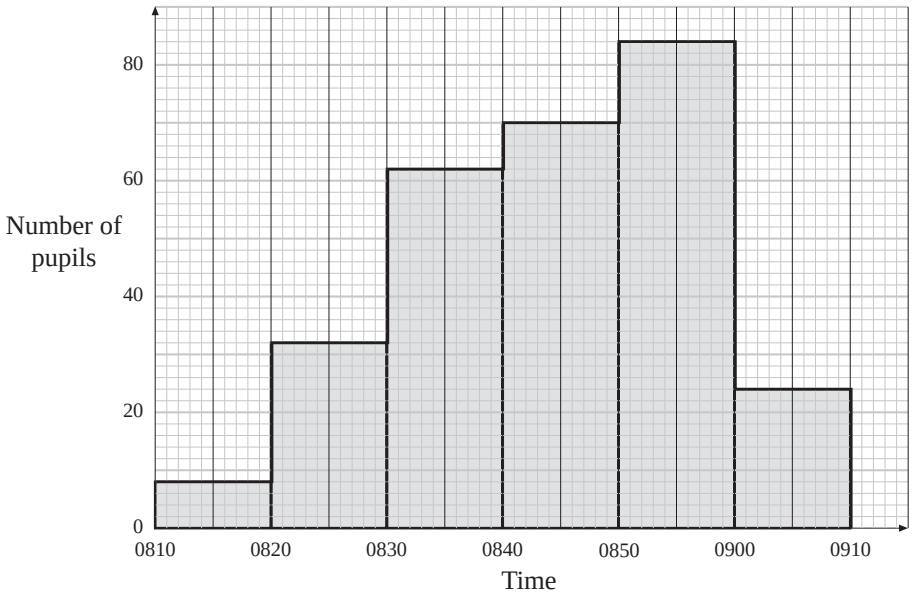
Score	$0 \le s < 20$	$20 \le s < 40$	$40 \le s < 60$	$60 \le s < 80$	$80 \le s < 100$
Frequency	82	166	342	220	54

Draw a histogram for the data.

Investigation

Choose an article of at least 500 words from a newspaper. Record the word lengths (in terms of number of letters in a word) of 500 successive words in the article. Tabulate the results in a frequency distribution. Find the mean, mode and median for the word lengths used.

11. The graph shows the result of a survey of the times at which pupils arrive at school one day.



How many pupils arrived for school between 0830 and 0850? *(SEG)*

12. The table below gives information about the expected lifetimes, in hours, of 200 light bulbs.

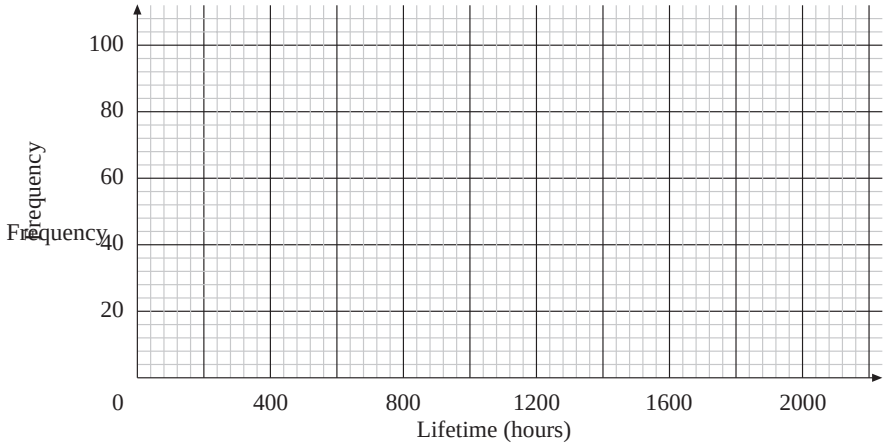
Lifetime (t)	$0 < t \leq 400$	$400 < t \leq 800$	$800 < t \leq 1200$	$1200 < t \leq 1600$	$1600 < t \leq 2000$
Frequency	32	56	90	16	6

(a) Mr Jones buys one of the light bulbs.

(i) What is the probability that it will not last more than 400 hours?

(ii) What is the probability that it will last at least 800 hours but not more than 1600 hours?

(b) Using axes similar to those below, draw a frequency polygon to illustrate the information in the table.



(MEG)

13. The height of some pupils is recorded.

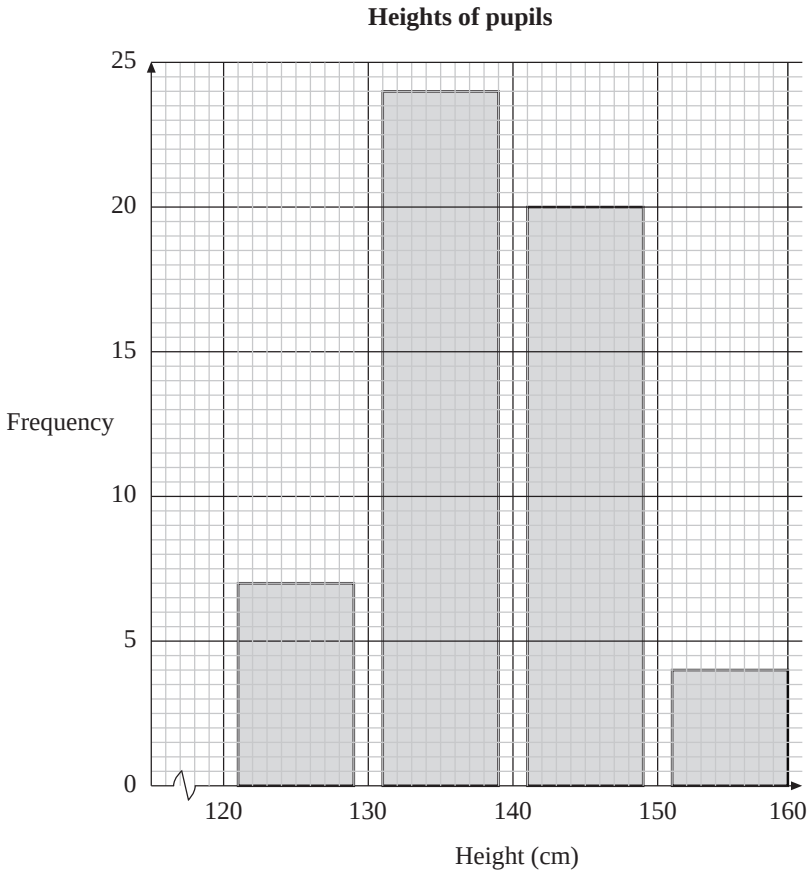
Height h (cm)	Frequency
$120 \le h < 125$	2
$125 \le h < 130$	5
$130 \le h < 135$	8
$135 \le h < 140$	14
$140 \le h < 145$	11
$145 \le h < 150$	9
$150 \le h < 155$	3
$155 \le h < 160$	1

Ann records the data using class intervals of 10 cm.

(a) Copy and complete Ann's table.

Height h (cm)	Frequency
$120 \le h < 130$	
$130 \le h < 140$	
$140 \le h < 150$	
$150 \le h < 160$	

Ann draws a frequency diagram of her data.



Ann has made two mistakes in drawing her diagram.

(b) What are the two mistakes?

Another pupil is included.

The pupil has a height of 150 cm.

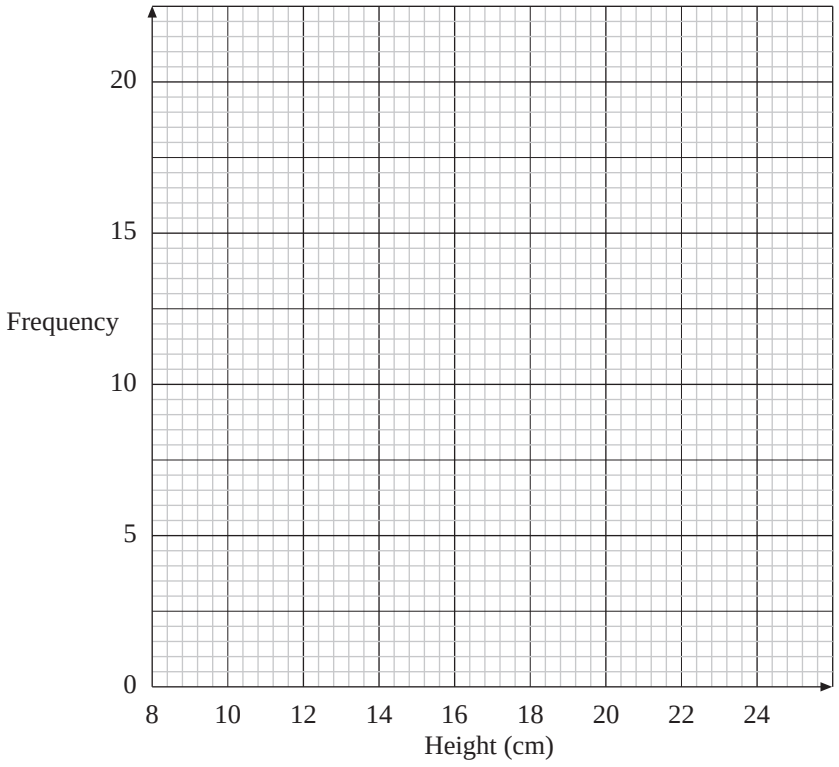
(c) Into which of Ann's class intervals should the pupil be placed?

(SEG)

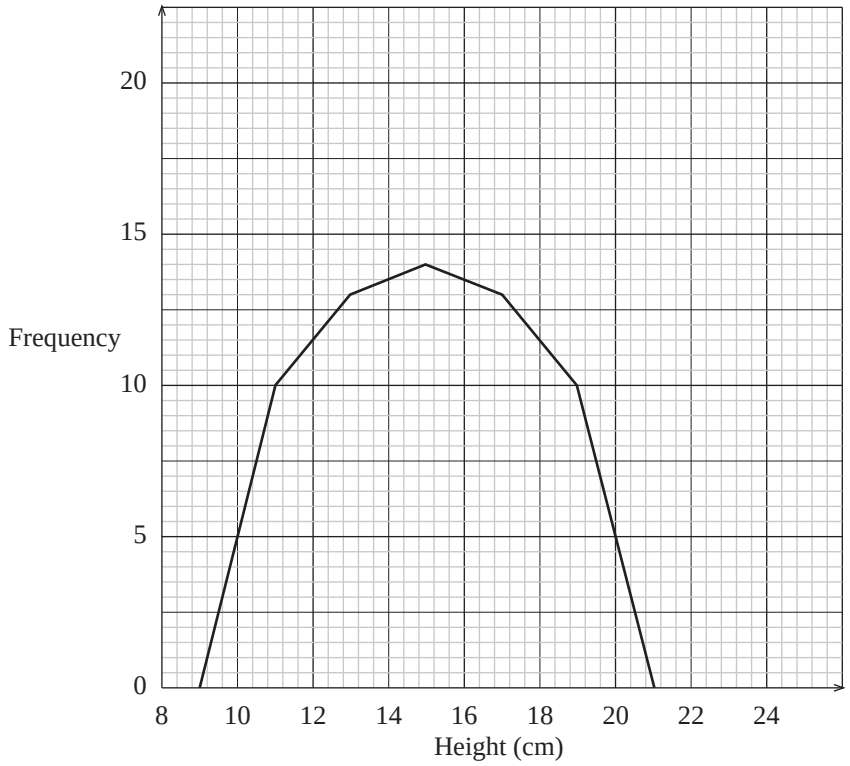
14. The height of each of 60 plants of type *A* was measured and recorded.

Height of plant (cm)	8—	10—	12—	14—	16—	18—	20-22
Number of plants	0	2	3	18	19	18	0

(a) Draw the frequency polygon of these results on a grid like the one below.



The following graph shows a frequency polygon of 60 plants of type *B*.



(b) Write down **two** differences between the two types of plant shown by the frequency polygons.

(SEG)

8.7 Histograms with Unequal Class Intervals

When drawing histograms it is possible that the intervals will not have the same width.

Consider the data given in the table below.

Weight (in grams)	*Frequency*
$0 \le w < 40$	5
$40 \le w < 50$	6
$50 \le w < 60$	8
$60 \le w < 70$	4
$70 \le w < 100$	2

The way the data have been presented makes it impossible to draw a histogram with equal class intervals.

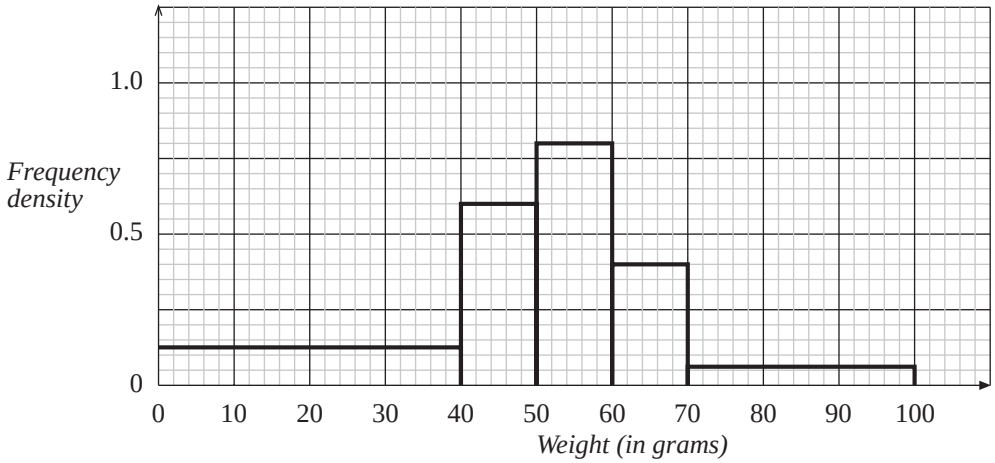
In order to keep the histogram fair the *area* of the bars, rather than the height, must be proportional to the frequency. So on the vertical scale we plot *frequency density* instead of frequency, where

$$\text{Frequency Density} = \frac{\text{Frequency}}{\text{Class Width}}$$

Rewriting the table with an extra column for frequency density, gives

Weight (in grams))	*Frequency*	*Frequency Density*
$0 \le w < 40$	5	$\frac{5}{40} = 0.125$
$40 \le w < 50$	6	$\frac{6}{10} = 0.6$
$50 \le w < 60$	8	$\frac{8}{10} = 0.8$
$60 \le w < 70$	4	$\frac{4}{10} = 0.4$
$70 \le w < 100$	2	$\frac{2}{30} = 0.067$

and you can draw the histogram with frequency density on the vertical axis.



Note

You can see that it is the area that is proportional to the frequency – in fact, a frequency of 1 is represented by 10 little squares.

Worked Example 1

Police officers recorded the speeds of vehicles passing a speed camera on an open road. Draw a histogram for this data.

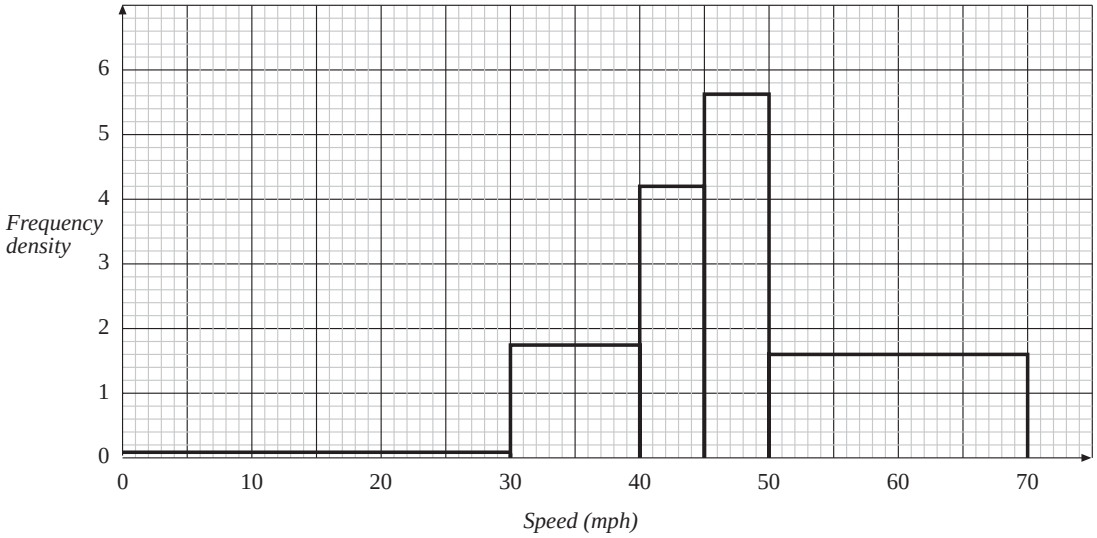
Speed (mph)	*Frequency*
$0 \le v < 30$	3
$30 \le v < 40$	17
$40 \le v < 45$	21
$45 \le v < 50$	28
$50 \le v < 70$	32

Solution

The following table shows how the frequency density can be calculated.

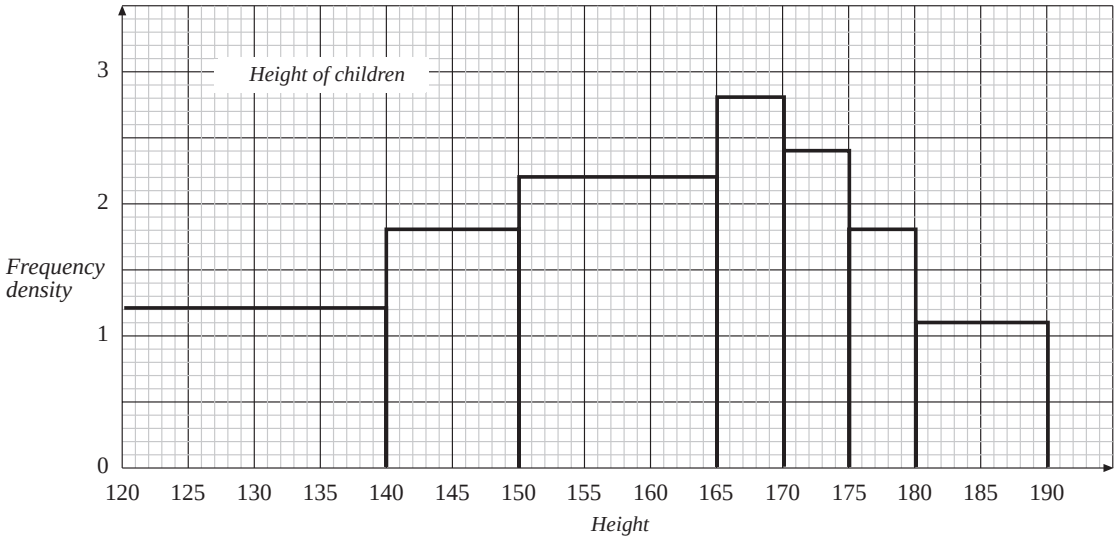
Speed (mph)	*Class width*	*Frequency*	*Frequency Density*
$0 \le v < 30$	30	3	$\frac{3}{30} = 0.1$
$30 \le v < 40$	10	17	$\frac{17}{10} = 1.7$
$40 \le v < 45$	5	21	$\frac{21}{5} = 4.2$
$45 \le v < 50$	5	28	$\frac{28}{5} = 5.6$
$50 \le v < 70$	20	32	$\frac{32}{20} = 1.6$

The histogram is now shown below.



Worked Example 2

The histogram below shows the results of a survey into the height of children in a school.



(a) Find the number of children with heights between:

(i) 120 and 140 cm,

(ii) 170 and 175 cm.

(b) Find the total number of children measured.

Solution

(a) (i) For the 120 to 140 cm interval:

$$\text{Frequency Density} = 1.2$$
$$\text{Class Width} = 20$$

$$\text{Frequency} = 20 \times 1.2$$
$$= 24 \text{ children}$$

(ii) For the 170 to 175 cm interval:

$$\text{Frequency Density} = 2.4$$
$$\text{Class Width} = 5$$

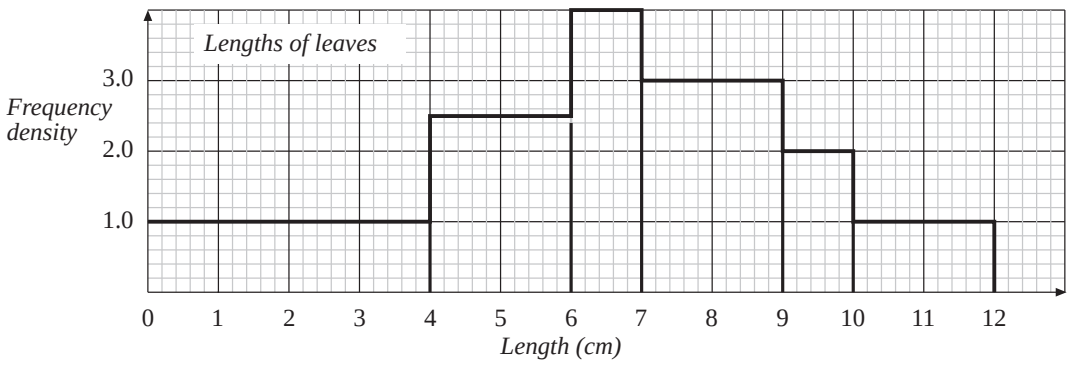
$$\text{Frequency} = 5 \times 2.4$$
$$= 12 \text{ children}$$

(b) To find the total, the numbers in each class interval must be found and added together.

$$\text{Total} = 20 \times 1.2 + 10 \times 1.8 + 15 \times 2.2 + 5 \times 2.8 + 5 \times 2.4 + 5 \times 1.8 + 10 \times 1.1$$
$$= 24 + 18 + 33 + 14 + 12 + 9 + 11$$
$$= 121$$

Exercises

1. For a project in Biology Sharma gathered data on the length of leaves from a tree and drew the histogram below.

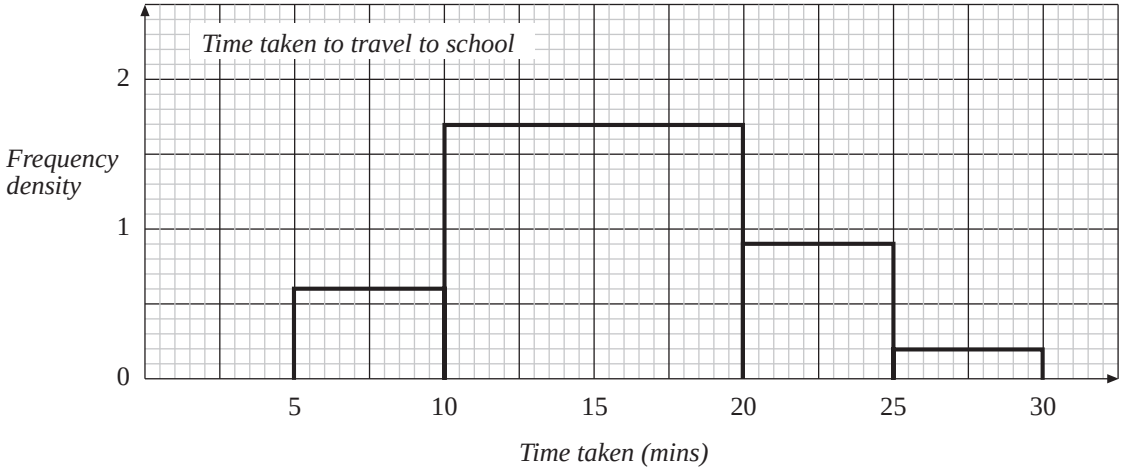


(a) How many leaves had a length less than 6 cm?

(b) How many leaves had a length greater than 9 cm?

(c) How many leaves did she measure?

2. Jennifer collected data on the length of time it took her to travel to school. She drew the histogram below.



This histogram contains an error. What is it?

3. Fred often travels by train. He kept a record of the time he had to wait when telephoning his local railway station to enquire about train times.

Waiting time (mins)	*Frequency*
$0 < t \leq 0.5$	8
$0.5 < t \leq 1.0$	10
$1.0 < t \leq 3.0$	15
$3.0 < t \leq 4.0$	12
$4.0 < t \leq 5.0$	3

Draw a histogram to show this data.

4. A teacher recorded all the scores of the students who took a maths test in his school. These scores are summarised in the table below.

Score	*Frequency*
$0 \leq t < 35$	3
$35 \leq t < 40$	7
$40 \leq t < 55$	20
$55 \leq t < 70$	42
$70 \leq t \leq 100$	16

Draw a histogram for this data.

5. A survey was carried out to find the weekly income of a group of Year 11 pupils. The income includes pocket money and wages from part-time jobs.

Income (£)	*Frequency*
$0 < I \leq 10$	8
$10 < I \leq 15$	24
$15 < I \leq 20$	19
$20 < I \leq 40$	7

Draw a histogram to show this data.

6. The distribution of the ages of inhabitants of a village is shown in the table below.

Age	*Frequency*
0 - 4	10
5 - 9	12
10 - 19	19
20 - 39	36
40 - 59	30
60 - 64	9
65 - 79	11
80 - 99	3

(a) Explain why the width of the first class interval is 5.

(b) Find the width of all the other class intervals.

(c) Draw a histogram to show this data.

7. The finishing times to the nearest minute for the competitors in a half-marathon to complete the race are given below.

135	103	123	116	177	101
107	117	126	117	156	133
127	95	134	110	201	115
202	113	170	105	132	105
155	98	137	115	129	117
152	93	116	92	152	97
169	112	163	124	151	143
160	121	176	100	84	114
122	98	96	118	153	112
153	136	123	80	170	109

Group the data into suitable intervals and draw a histogram.

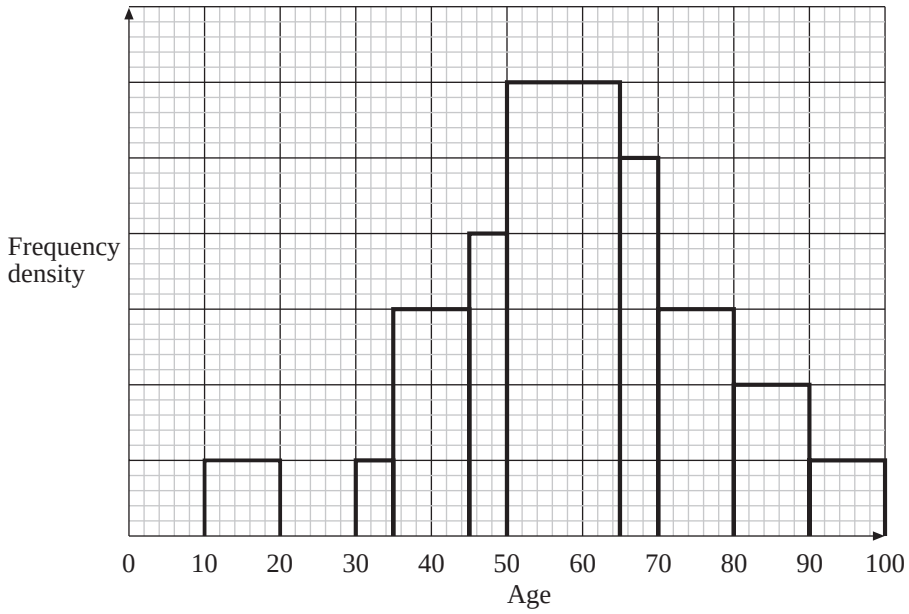
8. A GCSE examiner recorded the time that it took to mark the essays students wrote in an exam. The times are in this table.

Time (mins)	*Frequency*
10 - 12	9
13 - 18	24
19 - 20	21
21 - 30	16
31 - 45	3

(a) State the width of each class interval.

(b) Draw a histogram to show this data.

9. The age of each person in a coach party is illustrated in the histogram below.



There are 6 people in the 70-80 age range.

(a) How many people are there in the 45-50 age range?

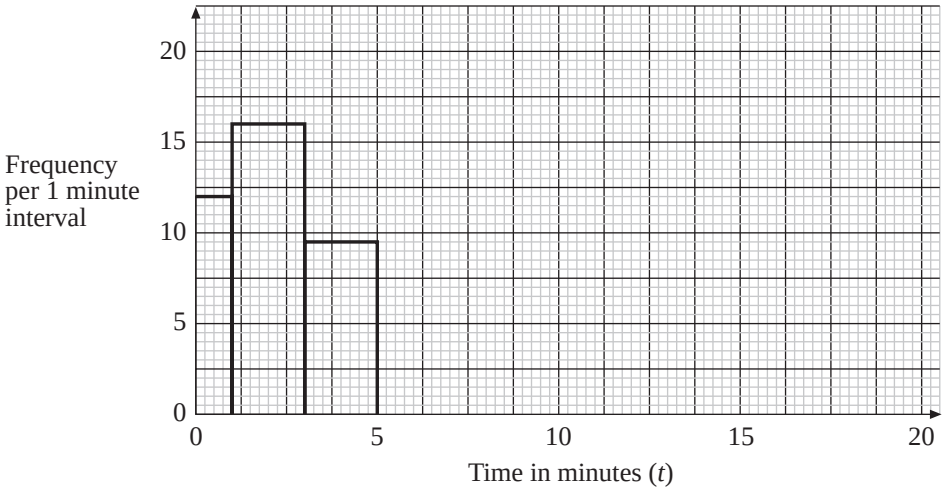
(b) How many people are there in the 50-70 age range?

(SEG)

10. A sample was taken of the telephone calls to a school switchboard. The lengths of the telephone calls are recorded, in minutes, in this table.

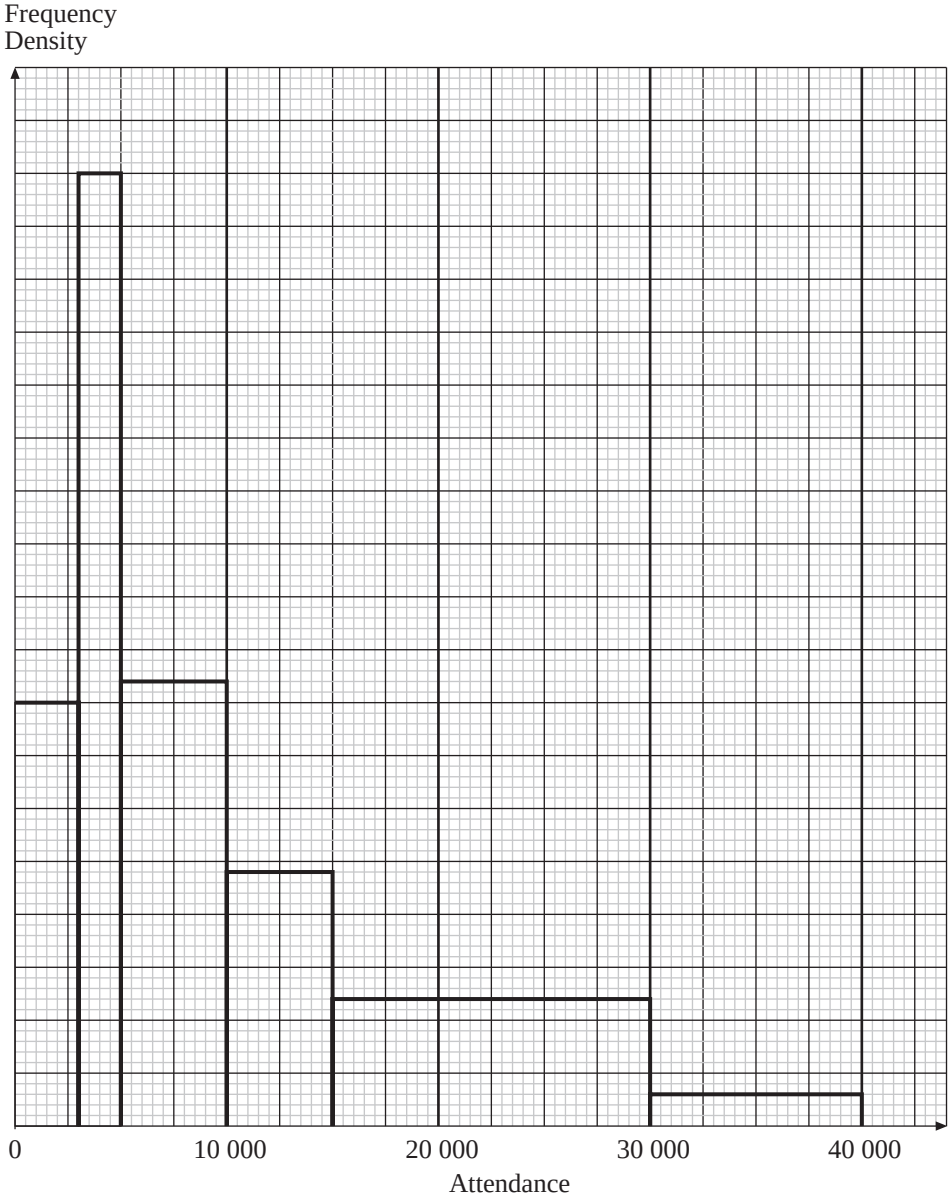
Time in minutes (t)	$0 < t \leq 1$	$1 < t \leq 3$	$3 < t \leq 5$	$5 < t \leq 10$	$10 < t \leq 20$
Number of calls	12	32	19	20	15

Copy and complete the histogram to show this information.



(MEG)

11. The histogram below represents the number of spectators at professional football matches one Saturday.



No match had more than 40 000 spectators.

At 4 matches the number of spectators was greater than or equal to 10 000 and less than 15 000.

(a) Use the information in the histogram to complete a copy of the frequency table below.

Number of spectators (n)	Frequency
$0 \leq n < 3000$	
$3000 \leq n < 5000$	
$5000 \leq n < 10\,000$	
$10\,000 \leq n < 15\,000$	4
$15\,000 \leq n < 30\,000$	
$30\,000 \leq n < 40\,000$	

(b) Calculate the total number of professional football matches played in England on that Saturday.

(LON)

8.8 Sampling

When conducting a survey it is often impossible to ask every individual who might be concerned or involved. For example, for a political opinion poll it is only possible to ask a *sample of the population* how they would vote.

The term *population* can be any group about which information is required. For example, the following could be populations:

Manchester United supporters,

kettles produced in a factory,

adult voters in the UK,

all pupils in your school.

Often conclusions are reached by looking at a sample taken from a population. There are three main methods for selecting a sample from a population:

Random Sample

The sample is formed by selecting members of the population at random. It is important to make sure that each member of the population is equally likely to be selected. Tables of random numbers can be used to help this process but more mundane methods, such as using numbers from a telephone book or even choosing a number from a hat of numbers, can be used. Scientific calculators also provide you with random numbers.

Systematic Sample

This type of sample is formed by taking members of the population at regular intervals. For example, by selecting every 5th or every 10th or every 12th member of the population.

Stratified Sample

The population is split into a number of groups. Random samples are then taken from each group so that the ratio of the *sizes* of the sample is the same as the ratio of the *number of members* of the groups in the population. For example, if a population contains 1000 women and 500 men, a stratified sample of total size 75 would contain 50 women and 25 men.

Worked Example 1

There are 12 teachers in a small school. A sample of size 4 is to be selected from this population.

1.	Mrs Skirton
2.	Mrs Pearson
3.	Mrs Green
4.	Mrs Crocker
5.	Mrs Barnes
6.	Mr Vinner
7.	Mr Krishnan
8.	Mr Gadd
9.	Mr Hampson
10.	Mr Thompson
11.	Mr Mobey
12.	Mr Grainger

(a) Create a systematic sample for the population.

(b) Create a random sample for the population.

Solution

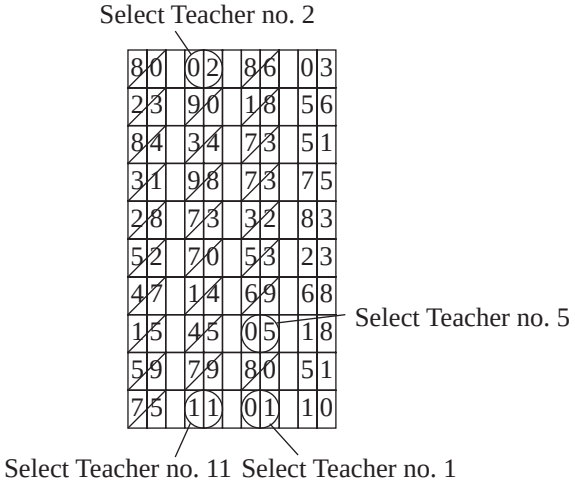
(a) As there are 12 teachers, a systematic sample could be made by selecting every third teacher from the list. This would create a sample containing:

Mrs Green, Mr Vinner, Mr Hampson and Mr Grainger.

8	0		0	2		8	6		0	3
2	3		9	0		1	8		5	6
8	4		3	4		7	3		5	1
3	1		9	8		7	3		7	5
2	8		7	3		3	2		8	3
5	2		7	0		5	3		2	3
4	7		1	4		6	9		6	8
1	5		4	5		0	5		1	8
5	9		7	9		8	0		5	1
7	5		1	1		0	1		1	0

(b) The list of random numbers shown here has been generated using a spreadsheet. The numbers have been arranged to give two digit numbers. Numbers greater than 12 are not needed in this case so these numbers can be deleted.

Beginning in the top left-hand corner and working down the columns gives the result shown below.



The teachers selected by this are:

No. 2	Mrs Pearson
No.11	Mr Mobey
No. 5	Mrs Barnes
No. 1	Mrs Skirton

Worked Example 2

A headteacher wishes to select a stratified sample of 50 pupils from Years 10, 11, 12 and 13. The table shows how many students are in each year.

Year	*Number of pupils*
10	320
11	300
12	180
13	150

Solution

First find the total number of pupils in the population.

$$\text{Total} = 320 + 300 + 180 + 150$$
$$= 950$$

The fraction of the population in Year 10 is $\frac{320}{950}$.

So the number of pupils selected from Year 10 is given by:

$$\frac{320}{950} \times 50 = 16.84, \text{ so select 17 pupils.}$$

The number of pupils selected from Year 11 is given by:

$$\frac{300}{950} \times 50 = 15.79, \text{ so select 16 pupils.}$$

The number of pupils selected from Year 12 is given by:

$$\frac{180}{950} \times 50 = 9.47, \text{ so select 9 pupils.}$$

The number of pupils selected from Year 13 is given by:

$$\frac{150}{950} \times 50 = 7.89, \text{ so select 8 pupils.}$$

Exercises

1. A factory contains 24 identical machines which are labelled:

   A, B, C, D, E, F, G, H, I, J, K, L, M, N, O, P, Q, R, S, T, U, V, W, X

   (a) Create a systematic sample containing:

   (i) 8 machines, (ii) 6 machines,

   (iii) 4 machines.

   (b) Use the random numbers in the table to create random samples containing:

   (i) 5 machines, (ii) 10 machines.

   (c) Describe another way that a random sample could be made.

6	8		0	6		4	2		3	8		0	6		1	5
5	0		9	6		2	4		2	1		8	2		5	3
5	5		4	0		3	2		4	6		7	4		7	6
0	6		5	7		3	5		4	8		2	0		9	2
2	8		7	4		4	4		7	3		4	4		2	5
6	5		0	8		5	6		6	8		4	5		7	4
6	3		7	7		8	7		5	8		0	0		0	9
1	3		2	6		7	6		7	6		8	1		6	0
8	0		5	1		8	9		5	9		3	8		1	0
2	7		2	2		8	7		5	5		8	0		5	2

2. The table shows the number of students in each year group of a school. How many students should be selected from each year group to create a stratified sample of 80 students?

Year	*Number of students*
7	150
8	148
9	162
10	154
11	152
12	80
13	62

3. A company wants to form a stratified sample to discuss issues with the staff.

	Number employed
Managers	8
Supervisors	20
Administrators	12
Manual Staff	140
Delivery Staff	30

(a) How many of each type of employee should be included in a sample of size 20?

(b) If a stratified sample contains 8 manual staff, what would be the size of the complete sample?

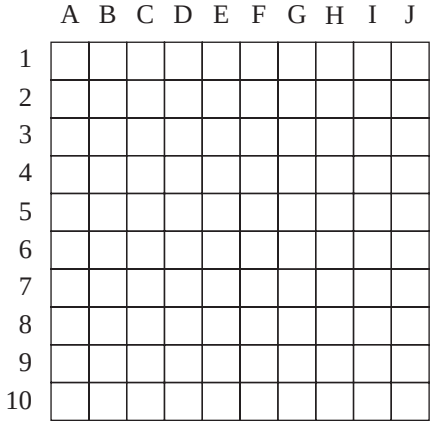
4. In order to form a sample a number of pupils are selected from a number of different teaching groups. The size of the samples and the teaching groups are given in the table below.

Group	*Number in Group*	*Number in Sample*
A	36	9
B	32	8
C	24	5
D	18	4

(a) Is this a stratified sample?

(b) How should it be changed to give a stratified sample?

5. For an experiment in Biology a square metre of ground has been divided into 100 squares as shown.



You are required to create a sample of 10 squares.

(a) Describe two ways of creating a systematic sample, giving the results from each method.

(b) Use the following list of random numbers to create a random sample.

1	1		5	7		4	2		7	0		4	1		1	2		8	8		5	1
3	2		3	8		1	4		6	5		4	9		4	6		1	8		6	2
4	8		2	0		4	7		4	4		5	6		6	5		4	6		3	6
4	2		5	9		6	1		4	1		7	8		4	2		7	2		1	1
5	0		4	2		5	8		7	8		7	1		7	8		1	2		3	7
6	6		0	8		2	1		8	4		9	4		6	1		3	1		3	0
1	4		6	4		5	1		0	5		5	3		9	3		4	5		8	6
9	3		4	9		0	5		2	7		5	4		1	8		6	4		5	7
8	7		6	1		5	5		6	7		2	3		2	6		7	0		7	5
0	3		8	7		1	9		4	8		1	0		6	9		3	5		6	1
4	2		5	2		8	3		7	4		3	5		0	9		1	3		3	6
5	1		4	3		7	6		6	2		9	1		3	9		8	9		7	5

Create a second random sample, by starting in a different place in the list of random numbers.

How many squares do the two samples have in common?

6. What problems might be encountered if samples are formed in the following ways?

   (a) Selecting people at random from a telephone directory.

   (b) Selecting every third person entering a shopping arcade.

   (c) Selecting people at random at a football match.

   (d) Selecting people leaving a golf course in red cars.

7. Describe a survey, in which each of the samples in question 6 could be a good sample to base the survey on.

8. Describe how you might select a sample if you were asked to conduct a survey to find out if:

   (a) the parents of primary school children were happy with their schools,

   (b) users of mobile phones were happy with the service provided,

   (c) the local bowling green was properly maintained,

   (d) the local public transport services are adequate.

9. (a) In a school there are 420 pupils in the lower school, 310 pupils in the middle school, and 130 pupils in the upper school.

      (i) How many pupils from each part of the school should be included in a stratified random sample of size 100?

      (ii) Explain why a stratified random sample should be taken rather than a simple random sample.

   (b) These 100 pupils were asked to keep a record of the number of hours of television that they watched in one week.

   The results are summarised below.

No. of hours watched	0–	5–	10–	20–	30–	40–	60–	80-100
No. of viewers	7	3	15	35	22	10	6	2

Draw a histogram to illustrate these data.

(SEG)