Linear Relationships

Solutions

Curriculum Ready

Mathletics

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Page 4 questions

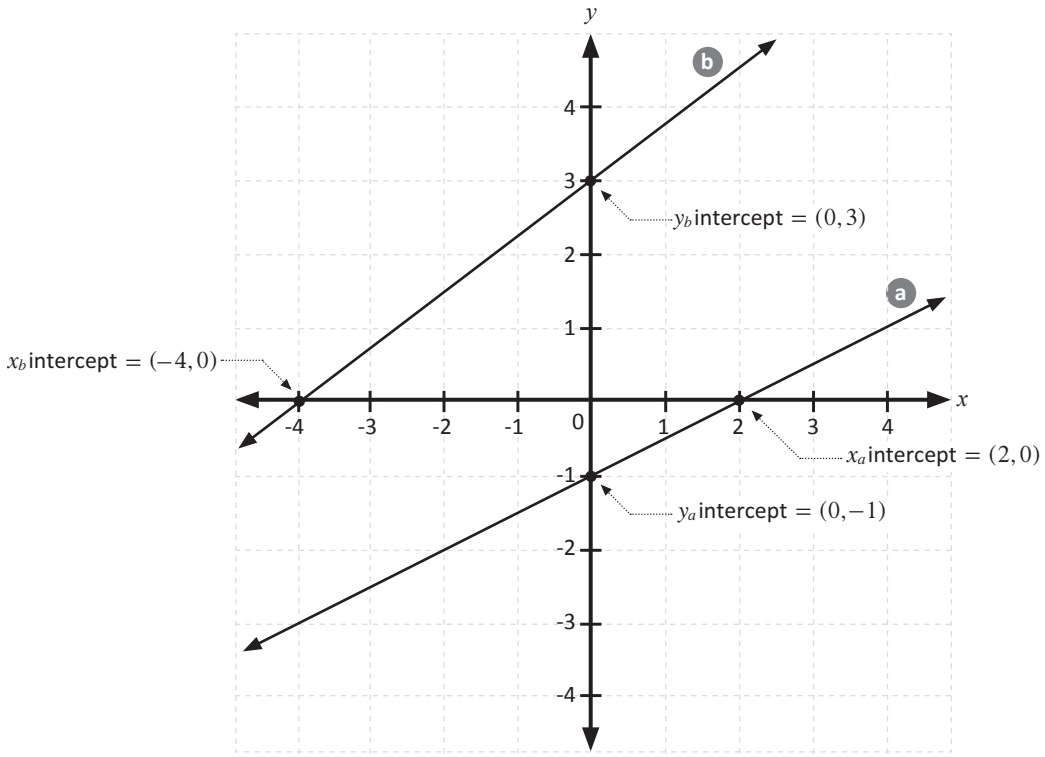
1. Draw the following lines on the provided axes:

a Line with x-intercept 2 and y-intercept -1.

The x-intercept is $(2, 0)$ and the y-intercept is $(0, -1)$

b Line with y-intercept 3 and x-intercept -4.

The y-intercept is $(0, 3)$ and the x-intercept $(-4, 0)$



2. Write the following in gradient x-intercept form:

Gradient x-intercept form is $y = mx + c$ form. Rearrange each equation into this form.

a $4x = 2y + 1$

$2y = 4x - 1$

$y = \dfrac{4x - 1}{2}$

$= \dfrac{4x}{2} - \dfrac{1}{2}$

$= \dfrac{4}{2}x - \dfrac{1}{2}$

$= 2x - \dfrac{1}{2}$

b $-y = x + 1$

$y = -x - 1$

Page 4 questions

c $x + 2y - 6 = 0$

$2y = -x + 6$

$y = \frac{-x}{2} + 3$

$y = -\frac{1}{2}x + 3$ or $y = -\frac{x}{2} + 3$

d $3x = -9y$

$9y = -3x + 0$

$y = \frac{-3}{9}x + 0$

$y = -\frac{1}{3}x$ or $y = -\frac{x}{3}$

Page 5 questions

3. Write in standard form the equation for a line with gradient $m = 3$ and y-intercept $b = 5$

Standard form is $y = mx + c$, where m is the gradient and c the y-intercept. From the question $m = 3$ and $c = 5$ so the standard form of the equation is $y = 3x + 5$.

4. Write the following in general form:

General form is $ax + bx + c = 0$. Rearrange each equation into this form:

a $y = 3x - 7$

$3x - 7 = y$

$3x - y - 7 = 0$

b $5x = 2y - 1$

$5x - 2y + 1 = 0$

c $y = 3 + \frac{x}{4}$

$3 + \frac{x}{4} = y$

$12 + x = 4y$

$x - 4y + 12 = 0$

d $-2x + 3y + 4 = 0$

$2x - 3y - 4 = 0$

5. Find the gradient of the line given by $12x + 4y = 8$ (Hint: Write in standard form first)

$12x + 4y = 8$

$4y = -12x + 8$

$y = -3x + 2$

Once the line is in standard form, the gradient (m) can be found by inspection and is 3 in this case.

6. Write the equation for a line with y-intercept $b = -2$ and gradient $m = 5$ in general form.

The gradient and y-intecept are given, so first write the equation in standard form ($y = mx + c$). Then rearrange into general form ($ax + by + c = 0$).

$y = 5x - 2$

$5x - 2 = y$

$5x - y - 2 = 0$

Page 7 questions

1. Which of the 2 points $(-1,6)$ or $(-1,5)$ lies on the line $y=-x+4$?

Both points have the same x-value of -1. Substitute the x-value into the equation of the line and see which point 'works' for the line.

$$\begin{aligned} y &= -x + 4 \\ y &= -(-1) + 4 \\ y &= 1 + 4 \\ y &= 5 \end{aligned}$$

The point $(-1,5)$ works for the equation, as the y-value produced by the equation matches the y-value of the point. It is then clear that the other point will not work, as we have just shown that $x=-1$ produces $y=5$ not $y=6$.

The point $(-1,5)$ is on the line $y=-x+4$.

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2. Find any possible values for x and y if the point (x,y) lies on the line $y=3x+7$.

Choose any x-value you like. I am choosing $x=100$.

$$\begin{aligned} y &= 3x + 7 \\ y &= 3(100) + 7 \\ y &= 300 + 7 \\ y &= 307 \end{aligned}$$

so the point $(100,307)$ works for the line $y=3x+7$.

We can check by substituting the values back into the equation.

$$\begin{aligned} y &= 3x + 7 \\ 307 &= 3(100) + 7 \\ 307 &= 300 + 7 \\ 307 &= 307 \end{aligned}$$

The point $(100,307)$ works for the line $y=3x+7$, which shows that the arithmetic is correct.

Page 8 questions

3. Find any possible values for x and y if the point (x, y) lies on the line $4y - 16x + 12 = 0$.

Choose any x-value you like. I am choosing $x = 2$. First we need to rearrange to make y the subject.

$$\begin{aligned} 4y - 16x + 12 &= 0 \\ 4y &= 16x - 12 \\ y &= \frac{16x - 12}{4} \\ y &= \frac{16x}{4} - \frac{12}{4} \\ y &= 4x - 3 \\ y &= 4(2) - 3 \\ y &= 8 - 3 \\ y &= 5 \end{aligned}$$

So $(2, 5)$ is a point on the line $4y - 16x + 12 = 0$.

We can check this by substituting the values back into the original equation:

$$\begin{aligned} 4y - 16x + 12 &= 0 \\ 4(5) - 16(2) + 12 &= 0 \\ 20 - 32 + 12 &= 0 \\ -12 + 12 &= 0 \\ 0 &= 0 \end{aligned}$$

Where we see that the equations 'work' for the point $(2, 5)$, so the arithmetic is correct.

4. Solve for x if the point $(x, 9)$ lies on the line $2y - 10x + 2 = 0$.

Rearrange the equation to make x the subject:

$$\begin{aligned} 2y - 10x + 2 &= 0 \\ 2y + 2 &= 10x \\ 10x &= 2y + 2 \\ x &= \frac{2y + 2}{10} \end{aligned}$$

and then substitute $y = 9$:

$$\begin{aligned} x &= \frac{2(9) + 2}{10} \\ x &= \frac{18 + 2}{10} \\ x &= \frac{20}{10} \\ x &= 2 \end{aligned}$$

So the point $(2, 9)$ is on the line $2y - 10x + 2 = 0$.

Page 9 questions

5. Are these lines parallel?

Parallel lines have the same gradient. Rewrite in standard form $(y = mx + c)$ and compare gradient (m) terms.

a $2x + 2y = 2$ and $2y = 2x + 3$

$$\begin{aligned} 2x + 2y &= 2 \\ 2y &= 2 - 2x \\ y &= -x + 1 \end{aligned}$$

$$\begin{aligned} 2y &= 2x + 3 \\ y &= x + \frac{3}{2} \end{aligned}$$

These lines are not parallel as the first gradient (-1) is different from the second (1).

b $y = 3x + 2$ and $y + 3x = 5$

$$y = 3x + 2$$

$$\begin{aligned} y + 3x &= 5 \\ y &= -3x + 5 \end{aligned}$$

These lines are not parallel as the first gradient (3) is different from the second (-3).

c $y = 2x - 3$ and $6x + 3y - 9 = 0$

$$y = 2x - 3$$

$$\begin{aligned} 6x + 3y - 9 &= 0 \\ 3y &= 9 - 6x \\ y &= -2x + 3 \end{aligned}$$

These lines are not parallel as the first gradient (2) is different from the second (-2).

d $y - 2x + 6 = 0$ and $4y = 8x + 1$

$$\begin{aligned} y - 2x + 6 &= 0 \\ y &= 2x - 6 \end{aligned}$$

$$\begin{aligned} 4y &= 8x + 1 \\ y &= 2x + \frac{1}{4} \end{aligned}$$

These lines are parallel as both gradients are the same (2).

Page 9 questions

6. Find the value of x if the line passing through $(5, 10)$ and $(x, 4)$ is parallel to $y = 6x + 7$.

Parallel lines have the same gradient.

The gradient of the $y = 6x + 7$ line is 6.

For the second line through $(5, 10)$ and $(x, 4)$, use the formula for the gradient of a line between two points and set this equal to 6.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$6 = \frac{10 - 4}{5 - x}$$

$$6(5 - x) = (10 - 4)$$

$$5 - x = \frac{10 - 4}{6}$$

$$5 - x = \frac{6}{6}$$

$$5 - x = 1$$

$$5 - 1 = x$$

$$x = 4$$

7. If a line has y-intercept 4 and is parallel to $y = -5x - 3$, then what is the equation of the line?

Let the new line be $y = mx + c$ where m is the gradient and c in the y-intercept.

m must be -5 to be parallel with $y = -5x - 3$. c must be 4 as the y-intercept is 4.

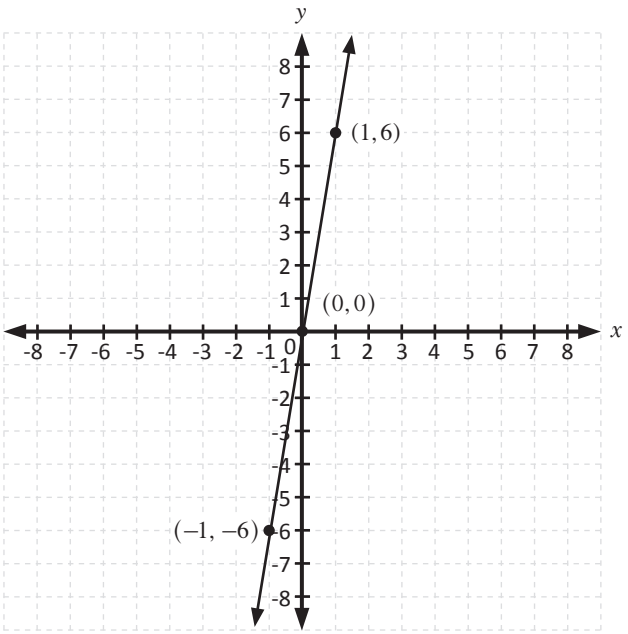
The equation of the line is then $y = -5x + 4$.

Page 13 questions

1. Draw the following graphs using the table method:

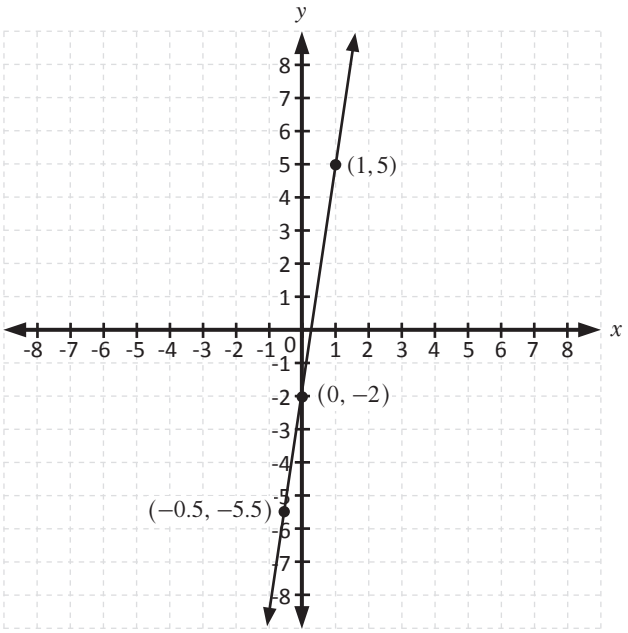
a $y = 6x$

x	-1	0	1
y	-6	0	6



b $y = 7x - 2$

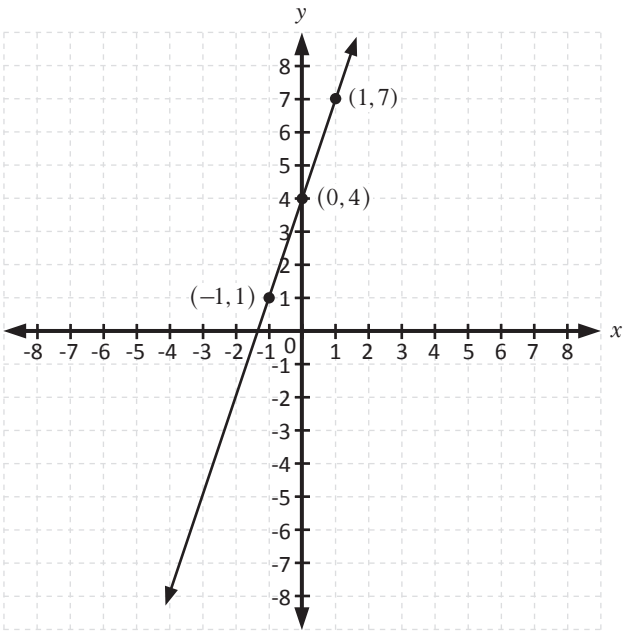
x	-0.5	0	1
y	-5.5	-2	5



Page 13 questions

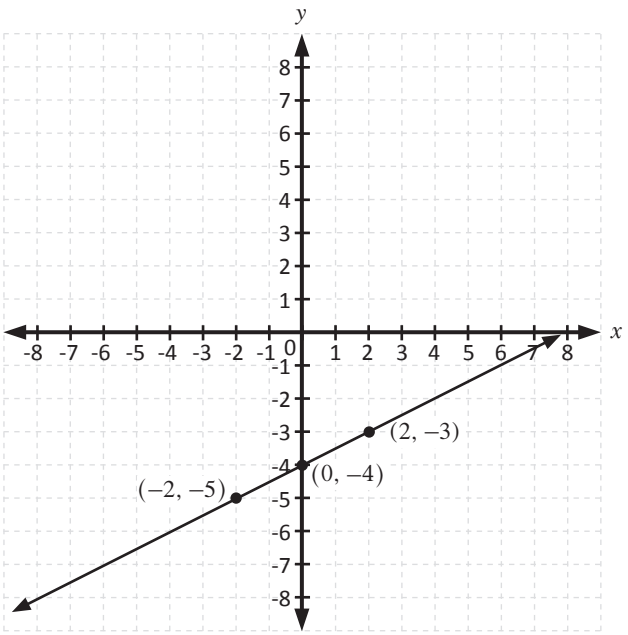
c $3x - y + 4 = 0$

x	-1	0	1
y	1	4	7



d $y = \frac{1}{2}x - 4$

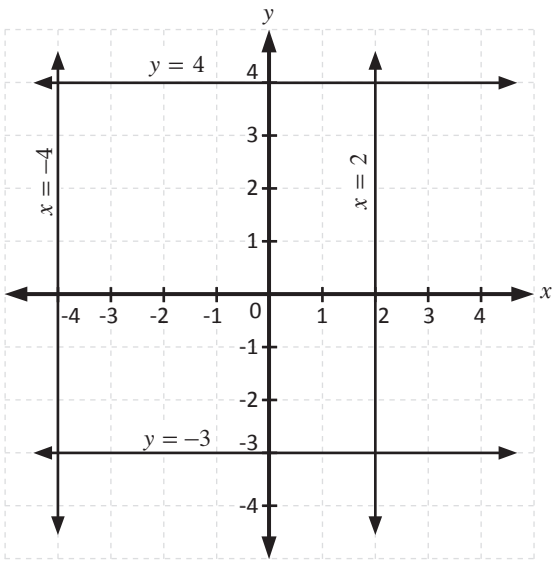
x	-2	0	2
y	-5	-4	-3



Page 14 questions

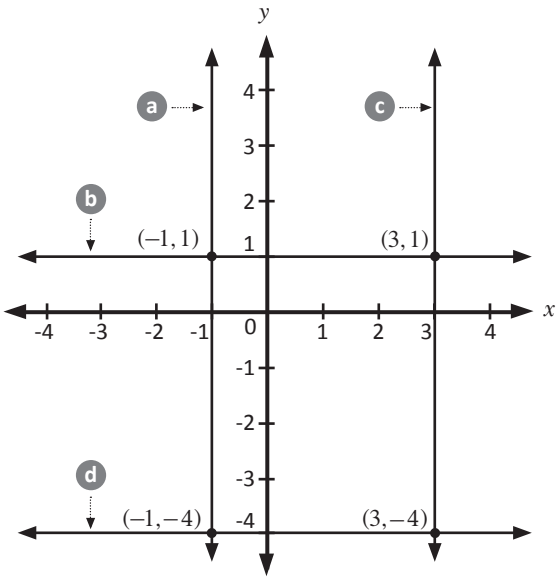
2. Draw the following lines on a number plane:

(a) $x = 2$ (b) $y = -3$ (c) $y = 4$ (d) $x = -4$



Page 15 questions

3. Write the equations of the following lines:



(a) $x = -1$ (b) $y = 1$ (c) $x = 3$ (d) $y = -4$

4. Write down the coordinates where lines (a) and (d) - from the above question - intersect each other.

(a) and (d) intersect at $(-1, -4)$

Page 15 questions

5. Find the equations of the following lines:

A vertical line passing through points $(-1, 5)$ and $(-1, -2)$:

$x = -1$

A horizontal line passing through $(0, 3)$:

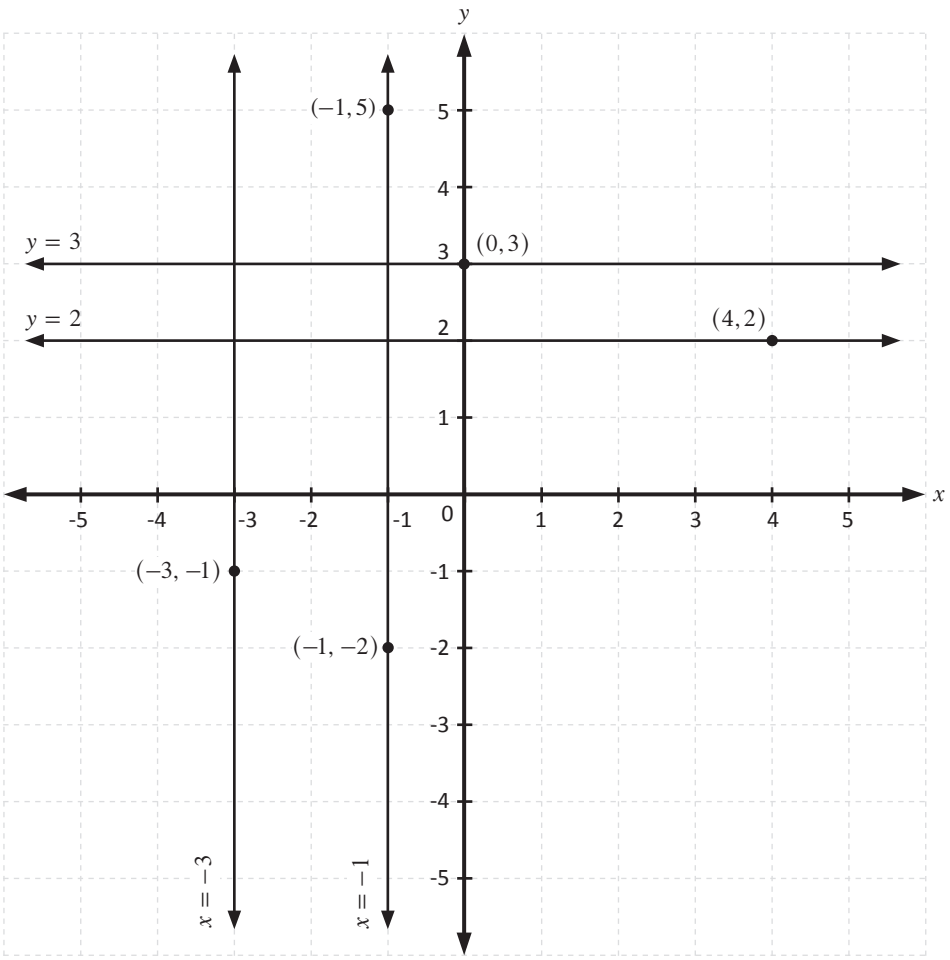
$y = 3$

A line parallel to the x-axis and passing through $(4, 2)$:

$y = 2$

A line parallel to the y-axis and passing through $(-3, 1)$:

$x = -3$



Page 16 questions

6. Draw the following graphs on the same set of axes:

a $y = -x$

x	-1	0	1
y	1	0	-1

b $y = -2x$

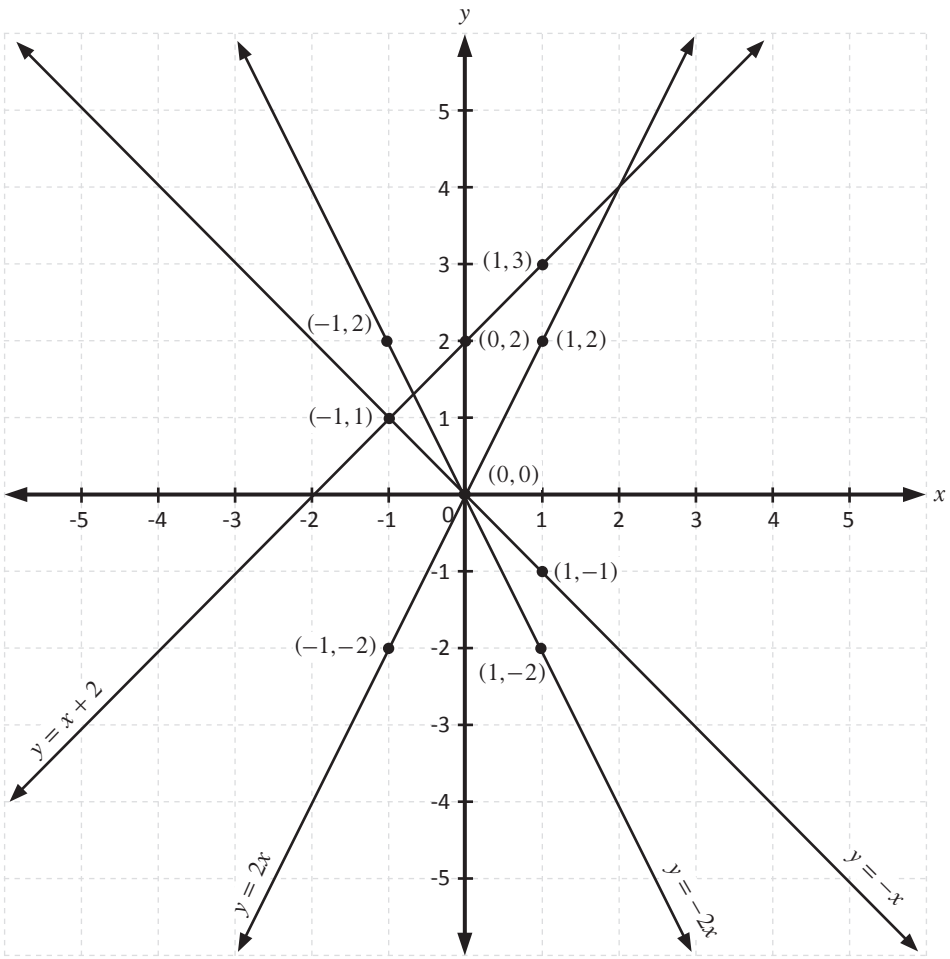
x	-1	0	1
y	2	0	-2

c $y = x + 2$

x	-1	0	1
y	1	2	3

d $y = 2x$

x	-1	0	1
y	-2	0	2



7. What do you notice about the lines as the value of m increases in their equations?

- If m is positive, the line moves from bottom left to top right
- If m is negative, the line moves from bottom right to top left

Page 17 questions

8. Draw these lines on the same set of axes below:

a $y = 6x$

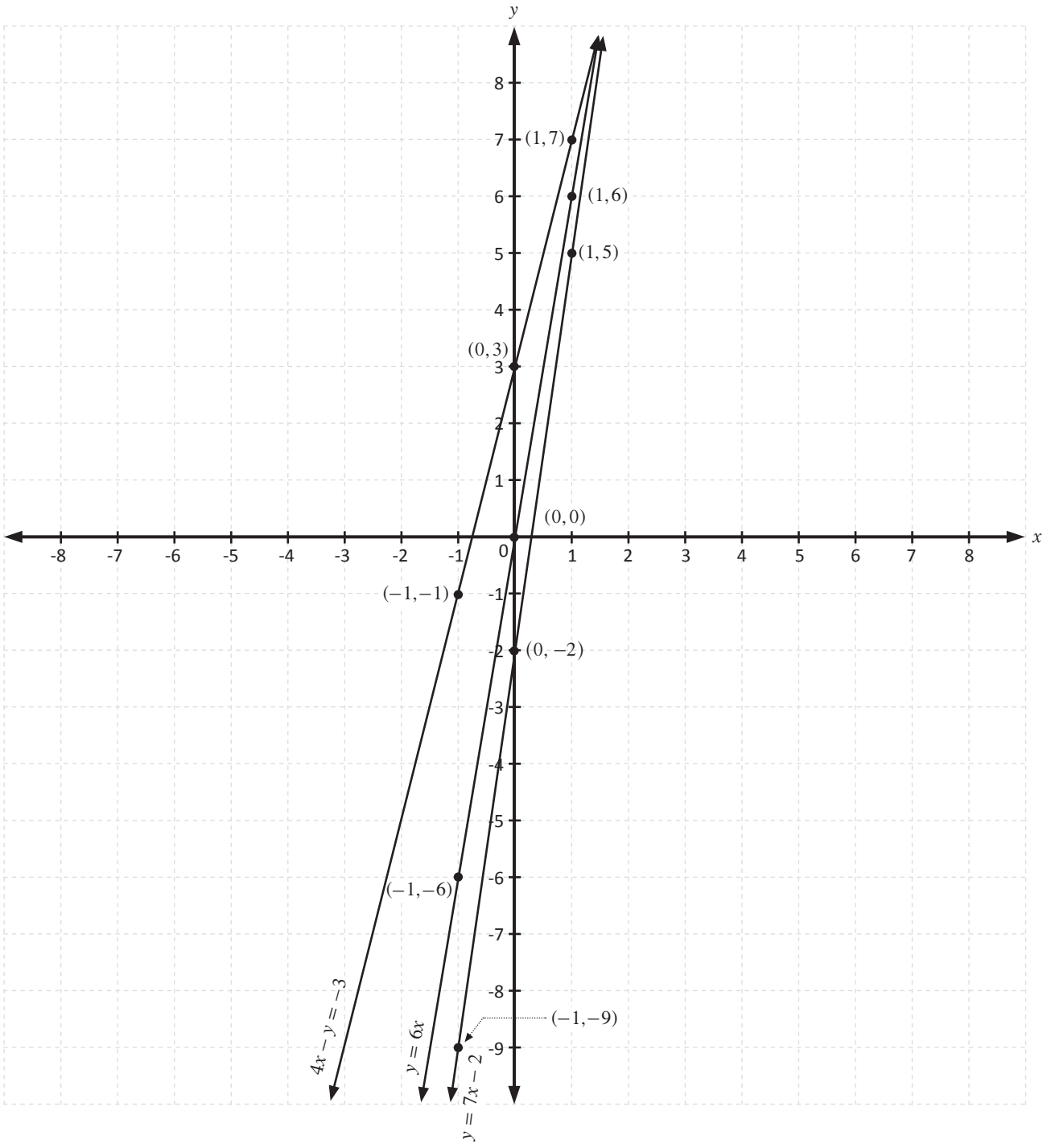
x	-1	0	1
y	-6	0	6

b $y = 7x - 2$

x	-1	0	1
y	-9	-2	5

c $4x - y = -3$

x	-1	0	1
y	-1	3	7



Page 21 questions

1. Find the intercepts of these lines:

a $y = 2x - 4$

This equation is in standard form $(y = mx + c)$ so the y-intercept is found by reading off the c value, -4.

Set $y = 0$ to find x-intercept:

$$\begin{aligned} y &= 2x - 4 \\ 0 &= 2x - 4 \\ 4 &= 2x \\ 2 &= x \\ x &= 2 \end{aligned}$$

The y-intercept is $(0,-4)$ and x-intercept is $(2,0)$.

b $x + y = -7$

Set $x = 0$ to find y-intercept:

$$\begin{aligned} x + y &= -7 \\ 0 + y &= -7 \\ y &= -7 \end{aligned}$$

Set $y = 0$ to find x-intercept:

$$\begin{aligned} x + y &= -7 \\ x + 0 &= -7 \\ x &= -7 \end{aligned}$$

The x-intercept is $(-7,0)$ and y-intercept is $(0,-7)$.

c $2x - y + 18 = 0$

Set $x = 0$ to find y-intercept:

$$\begin{aligned} 2x - y + 18 &= 0 \\ 2(0) - y + 18 &= 0 \\ -y + 18 &= 0 \\ 18 &= y \\ y &= 18 \end{aligned}$$

Set $y = 0$ to find x-intercept:

$$\begin{aligned} 2x - y + 18 &= 0 \\ 2x - (0) + 18 &= 0 \\ 2x + 18 &= 0 \\ 2x &= -18 \\ x &= -9 \end{aligned}$$

The x-intercept is $(-9,0)$ and y-intercept is $(0,18)$.

Page 21 questions

d $x - 3y - 21 = 0$

Set $y = 0$ to find x-intercept:

$$x - 3(0) - 21 = 0$$
$$x = 21$$

Set $x = 0$ to find y-intercept:

$$0 - 3y - 21 = 0$$
$$3y = -21$$
$$y = -7$$

The x-intercept is $(21, 0)$ and y-intercept is $(0, -7)$.

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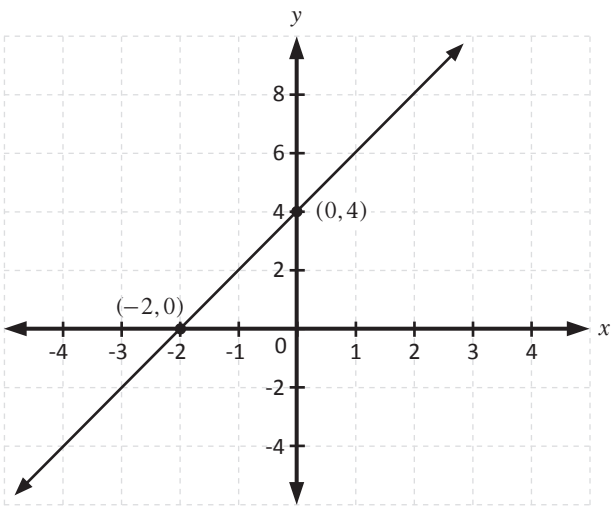
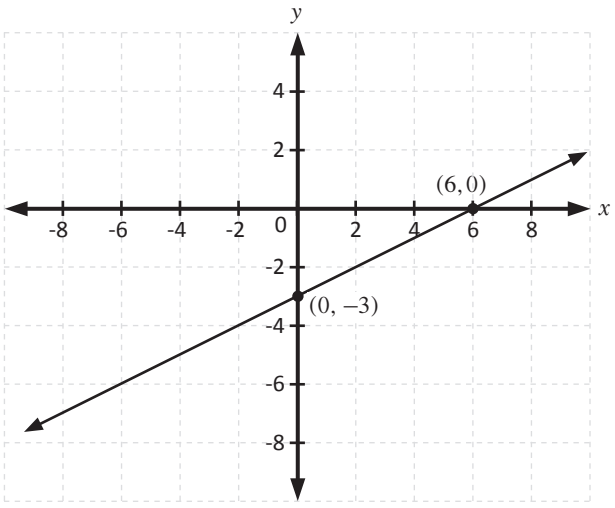
2. Draw the graph of a line with intercepts:

a x-intercept -2 and y-intercept 4



b x-intercept 6 and y-intercept -3



Page 22 questions

3. Use the intercept method to sketch the graphs of the following equations:

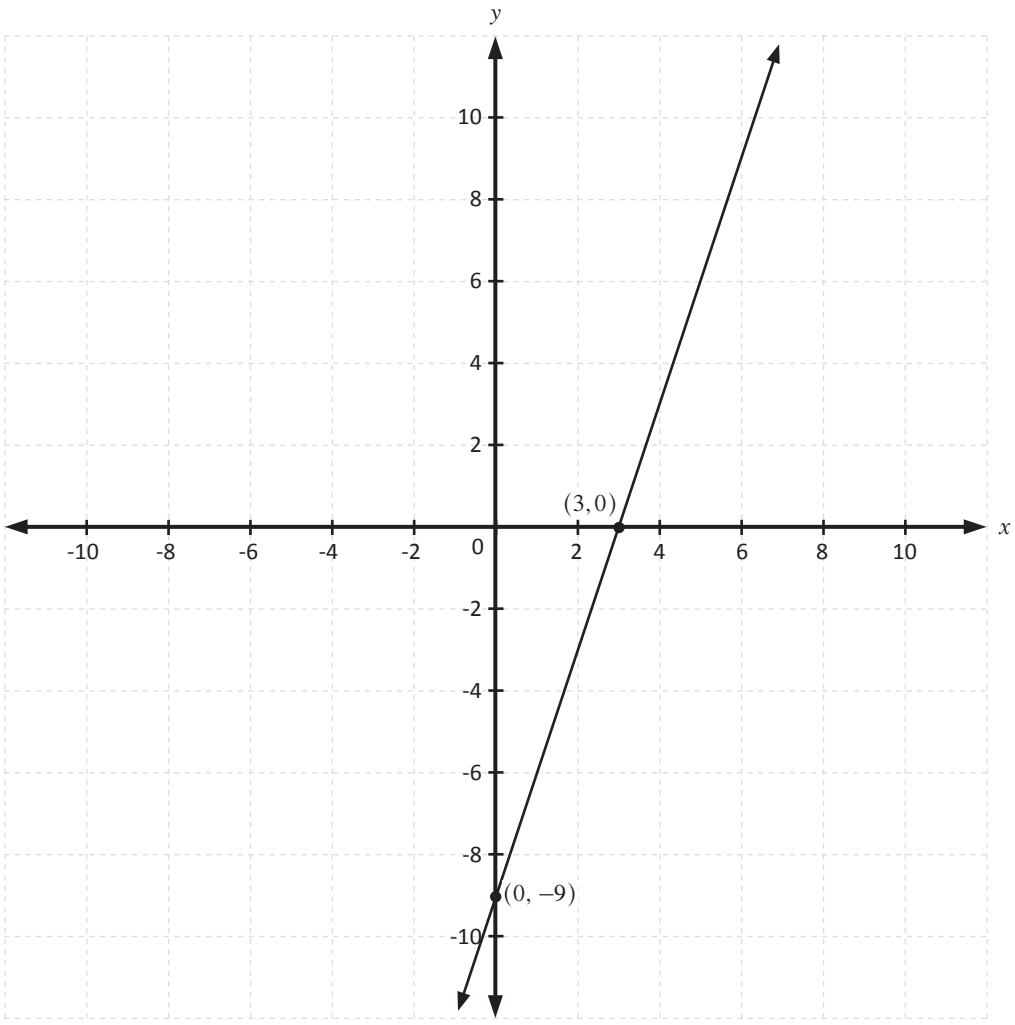
a $y = 3x - 9$

The equation is in $y = mx + c$ form so the y-intercept, -9 can be seen directly.

The x-intercept is found by setting $y = 0$ and rearranging to find x:

$$\begin{aligned} y &= 3x - 9 \\ 0 &= 3x - 9 \\ 9 &= 3x \\ 3x &= 9 \\ x &= 3 \end{aligned}$$

The x-intercept is at $(3, 0)$.



Page 22 questions

b $6y + 12x + 30 = 0$

Set $x = 0$ to find y-intercept:

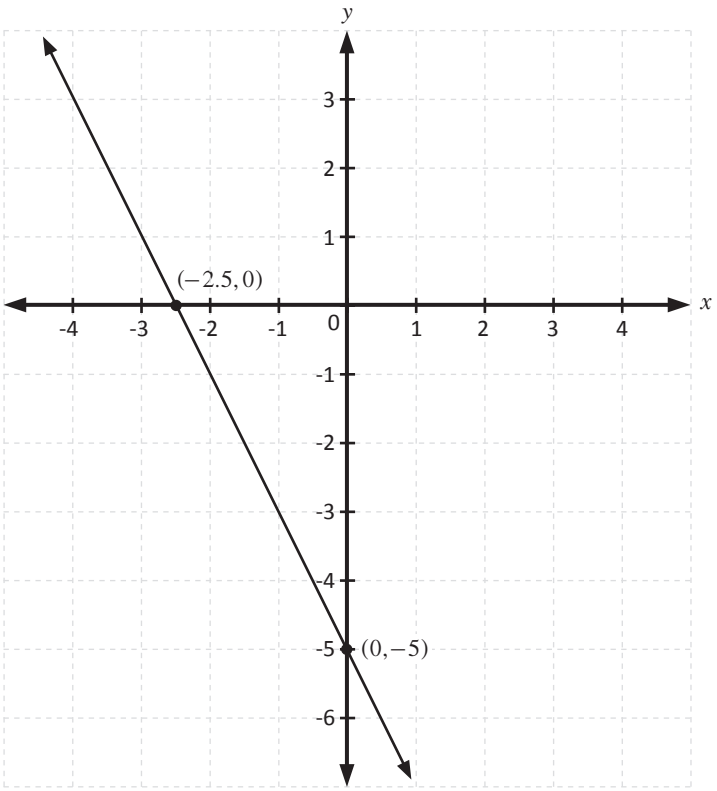
$$\begin{aligned} 6y + 12x + 30 &= 0 \\ 6y + 12(0) + 30 &= 0 \\ 6y + 30 &= 0 \\ 6y &= -30 \\ y &= -5 \end{aligned}$$

The y-intercept is at $(0, -5)$.

Set $y = 0$ to find x-intercept:

$$\begin{aligned} 6y + 12x + 30 &= 0 \\ 6(0) + 12x + 30 &= 0 \\ 12x + 30 &= 0 \\ 12x &= -30 \\ x &= -\frac{30}{12} \\ &= -\frac{5}{2} \\ &= -2\frac{1}{2} \\ &= -2.5 \end{aligned}$$

The x-intercept is at $(-2.5, 0)$.



Page 23 questions

4. Graph each pair of lines on the same axis to find the point of intersection:

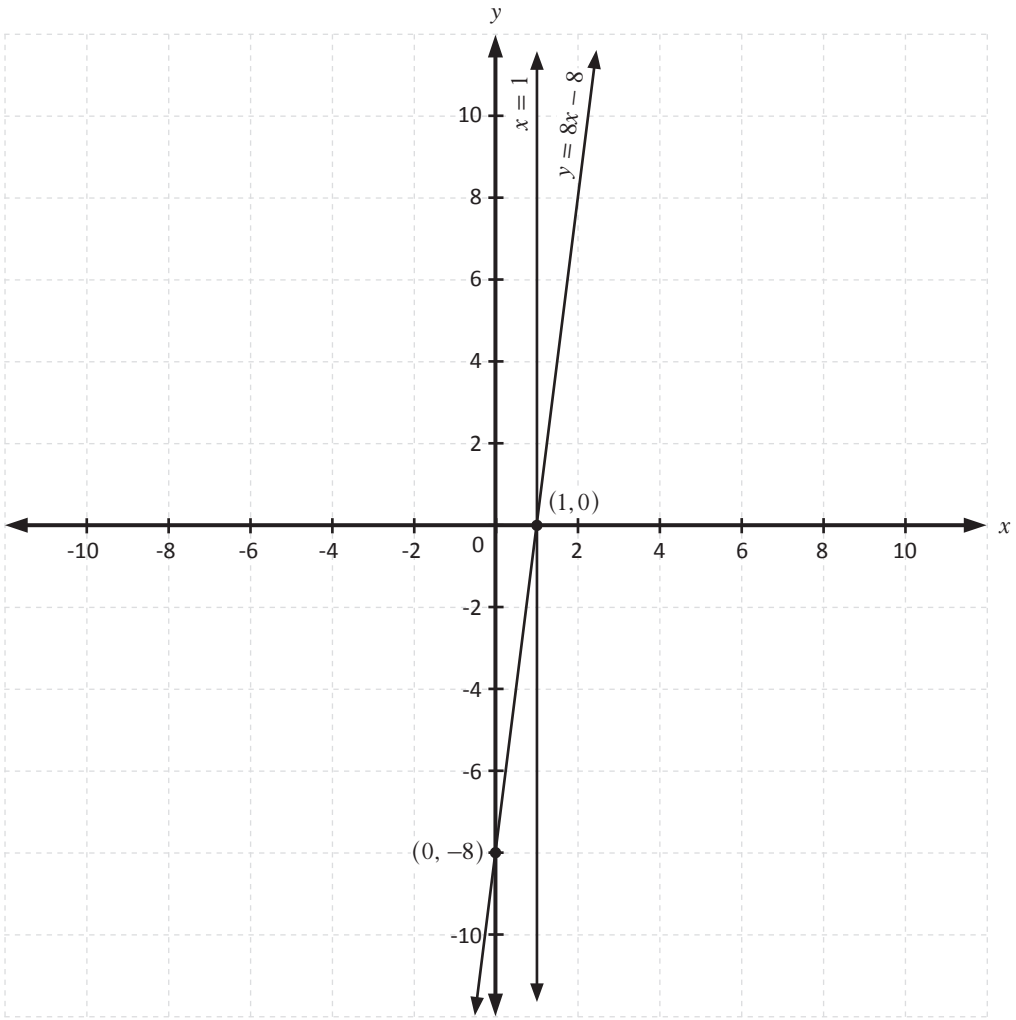
a $x = 1$ and $y = 8x - 8$

As the second line is in standard form we can see that the y-intercept is at $(0, -8)$.

Set $y = 0$ to find x-intercept:

$$\begin{aligned} y &= 8x - 8 \\ 0 &= 8x - 8 \\ 8 &= 8x \\ x &= 1 \end{aligned}$$

The x-intercept is at $(1, 0)$.



From the graph, the point of intersection is $(1, 0)$.

Page 23 questions

b $y = x - 5$ and $2x + y + 4 = 0$

Inspecting $y = x - 5$ shows the y-intercept is at $(0,-5)$.

Set $y = 0$ to find x-intercept:

$$\begin{aligned} y &= x - 5 \\ 0 &= x - 5 \\ 5 &= x \\ x &= 5 \end{aligned}$$

The x-intercept is at $(5,0)$.

For $2x + y + 4 = 0$ set $y = 0$ to find the x-intercept:

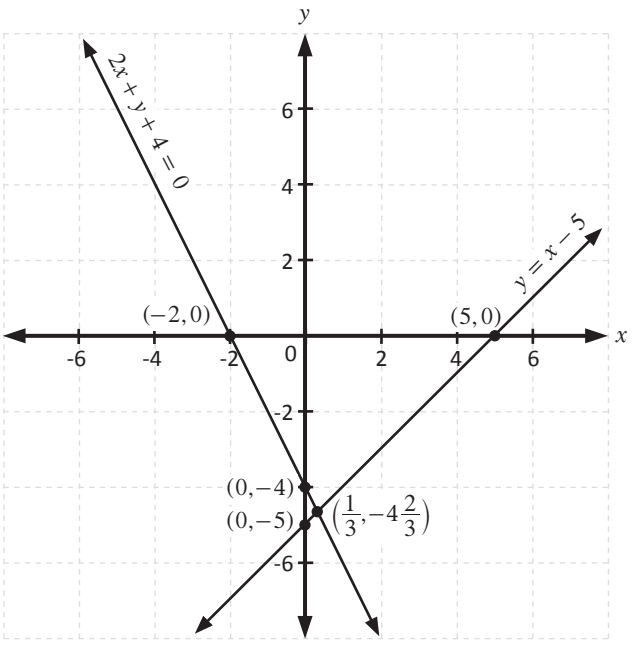
$$\begin{aligned} 2x + y + 4 &= 0 \\ 2x + 0 + 4 &= 0 \\ 2x + 4 &= 0 \\ 2x &= -4 \\ x &= -2 \end{aligned}$$

The x-intercept is at $(-2,0)$.

Set $x = 0$ to find y-intercept:

$$\begin{aligned} 2x + y + 4 &= 0 \\ 2(0) + y + 4 &= 0 \\ y + 4 &= 0 \\ y &= -4 \end{aligned}$$

The y-intercept is at $(0,-4)$.



From the graph, we can see the point of intersection is $(\frac{1}{3}, -4\frac{2}{3})$.

Page 23 questions

5. Find the point of intersection without drawing any graphs:

a $y = -2$ and $x - 2y - 11 = 0$

As y is already the subject of the first equation, substitute the first equation into the second and solve for x.

$$\begin{aligned} x - 2y - 11 &= 0 \\ x - 2(-2) - 11 &= 0 \\ x + 4 - 11 &= 0 \\ x - 7 &= 0 \\ x &= 7 \end{aligned}$$

From the first equation we know that $y = -2$, so the point of intersection is $(7, -2)$.

b $y = 5x - 8$ and $6x + 2y - 20 = 0$

As y is already the subject of the first equation, it is easy to substitute the first equation into the second, and solve for x.

$$\begin{aligned} 6x + 2y - 20 &= 0 \\ 6x + 2(5x - 8) - 20 &= 0 \\ 6x + 10x - 16 - 20 &= 0 \\ 16x - 36 &= 0 \\ x &= \frac{36}{16} \\ x &= \frac{9}{4} \end{aligned}$$

As y is already the subject of the first equation, substitute $x = \frac{9}{4}$ into the first equation to find y.

$$\begin{aligned} y &= 5x - 8 \\ y &= 5\left(\frac{9}{4}\right) - 8 \\ y &= \frac{45}{4} - 8 \\ y &= \frac{13}{4} \end{aligned}$$

The point of intersection is $\left(\frac{9}{4}, \frac{13}{4}\right)$.

Page 24 questions

6. Draw the following lines on the same set of axes:

- **Line 1:** $y = 3x + 6$
- **Line 2:** $y = 3x - 6$
- **Line 3:** $y = -2x + 8$

Find intercepts for each line. All three lines are in standard from, so the y-intercepts can be found by inspection.

Line 1: $y = 3x + 6$

The line 1 y-intercept is at $(0, 6)$.

Set $y = 0$ to find x-intercept:

$$\begin{aligned} y &= 3x + 6 \\ 0 &= 3x + 6 \\ -3x &= 6 \\ x &= \frac{6}{-3} \\ x &= -2 \end{aligned}$$

Line 1 x-intercept is at $(-2, 0)$.

Line 2: $y = 3x - 6$

The line 2 y-intercept is at $(0, -6)$.

Set $y = 0$ to find x-intercept:

$$\begin{aligned} y &= 3x - 6 \\ 0 &= 3x - 6 \\ 6 &= 3x \\ 3x &= 6 \\ x &= \frac{6}{3} \\ x &= 2 \end{aligned}$$

Line 2 x-intercept is at $(2, 0)$.

Line 3: $y = -2x + 8$

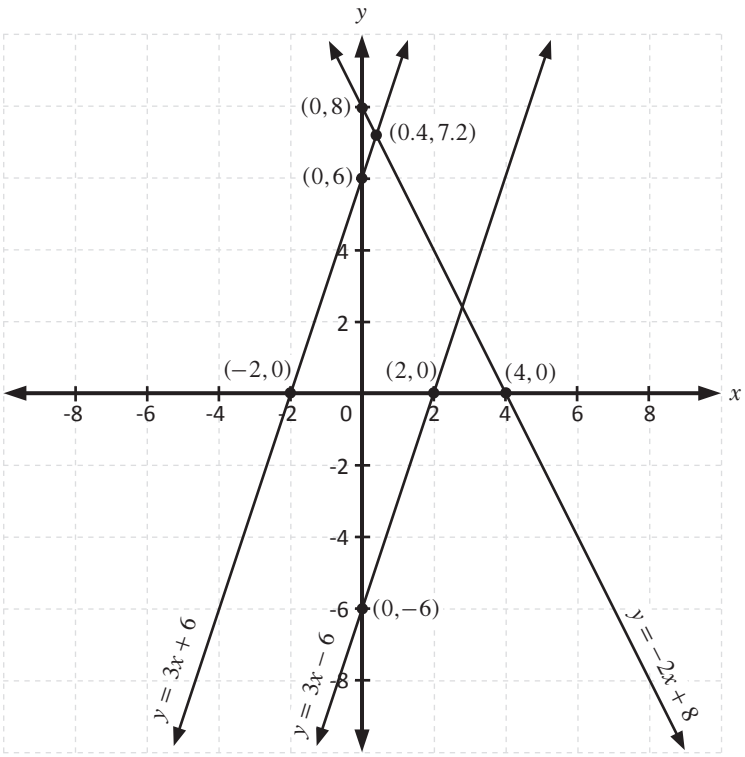
The line 3 y-intercept is at $(0, 8)$.

Set $y = 0$ to find x-intercept:

$$\begin{aligned} y &= -2x + 8 \\ 0 &= -2x + 8 \\ 2x &= 8 \\ x &= \frac{8}{2} \\ x &= 4 \end{aligned}$$

Line 3 x-intercept is at $(4, 0)$.

Page 24 questions



a **What is the point of interception of Line 1 and Line 3?**

Substitute Line 1 into Line 3 and solve for x:

$$\begin{aligned} y &= -2x + 8 \\ 3x + 6 &= -2x + 8 \\ 3x + 2x &= 8 - 6 \\ 5x &= 2 \\ x &= \frac{2}{5} \\ &= 0.4 \end{aligned}$$

Substitute this x-value into Line 3 to find y:

$$\begin{aligned} y &= -2x + 8 \\ &= -2(0.4) + 8 \\ &= -0.8 + 8 \\ &= 7.2 \end{aligned}$$

The point of intersection of Line 1 and Line 3 is $(0.4, 7.2)$.

b **Will Line 1 and Line 2 intersect at any point?**

No.

c **Why do you think this is so?**

Line 1 and Line 2 have the same gradient. This means they are parallel and will not intersect at any point.

Page 25 questions

7. Identify whether the following pairs of lines will intersect or not.

a $y = 4x + 2$ and $y = 4x - 7$

These have the same gradient (4) so they will not intersect.

b $y = 2x + 2$ and $x = -2x - 7$

These have different gradients $(2 \text{ and } -2)$ so they will intersect.

c $x + y = 7$ and $y = x + 2$

Change the first line to standard form so that the gradient can be seen:

$$\begin{aligned} x + y &= 7 \\ y &= 7 - x \\ y &= -x + 7 \end{aligned}$$

The gradients are different $(-1 \text{ and } 1)$ so the lines will intersect.

d $3x + 4y + 3 = 0$ and $6x + 8y + 5 = 0$

Rearrange both lines to standard form to see the gradients:

$$\begin{aligned} 3x + 4y + 3 &= 0 \\ 4y &= -3x - 3 \\ y &= \frac{-3x - 3}{4} \\ &= -\frac{3}{4}x - \frac{3}{4} \end{aligned}$$

$$\begin{aligned} 6x + 8y + 5 &= 0 \\ 8y &= -6x - 5 \\ y &= \frac{-6x - 5}{8} \\ &= \frac{-6}{8}x - \frac{5}{8} \\ &= -\frac{3}{4}x - \frac{5}{8} \end{aligned}$$

These lines have the same gradient $-\frac{3}{4}$ so they will not intersect.

Page 25 questions

8. Use substitution to prove that $y = \frac{1}{2}x + 13$ and $y = 4x - 1$ intersect at the point $(4, 15)$.

(Hint: Show the point of intersection is on both lines)

Substitute $x = 4$ into first line:

$$\begin{aligned} y &= \frac{1}{2}x + 13 \\ &= \frac{1}{2}(4) + 13 \\ &= 2 + 13 \\ &= 15 \end{aligned}$$

This shows the point $(4, 15)$ is on the first line.

Substitute $x = 4$ into the second line:

$$\begin{aligned} y &= 4x - 1 \\ &= 4(4) - 1 \\ &= 16 - 1 \\ &= 15 \end{aligned}$$

This shows the point $(4, 15)$ is on the second line.

If the same point is on 2 different lines, then it must be the intersection point of the lines.

9. What is the point of intersection of the lines $y = -3$ and $x = 17$?

The first line is horizontal with all y-value being -3. The second line is vertical with all x-values being 17, so the point of interaction is $(17, -3)$.

10. Find the equation of the horizontal line, exactly in the midle of $y = -4$ and $y = 6$.

The new line will also be horizontal, so it will be a $y =$ something line.

Use an average to find a number that is half way between -4 and 6:

$$\begin{aligned} \text{average} &= \frac{-4 + 6}{2} \\ &= \frac{2}{2} \\ &= 1 \end{aligned}$$

The line exactly in the middle of $y = -4$ and $y = 6$ is $y = 1$. You can check this by noting that 1 is 5 below 6, and 5 above -4.