1. (a) Factorize $x^2 - 3x - 10$.

(b) Solve the equation $x^2 - 3x - 10 = 0$.

Working:

$x^2 - 3x - 10$

$1 \quad x \quad -5$

$1 \quad x \quad 2$

$(x-5)(x+2)$

Answers:

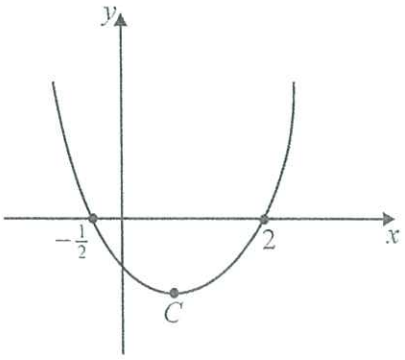
(a) $(x-5)(x+2)$

(b) $x = 5, -2$

(Total 4 marks)

2. The diagram represents the graph of the function

$$f: x \mapsto (x-p)(x-q).$$



(a) Write down the values of p and q.

(b) The function has a minimum value at the point C. Find the x-coordinate of C.

Working:

a) $f(x) = (x+\frac{1}{2})(x-2)$.

b) minimum is the midpoint between p and q.

$\therefore C = \frac{-\frac{1}{2}+2}{2} = \frac{\frac{3}{2}}{2}$

$= \frac{3}{4}$

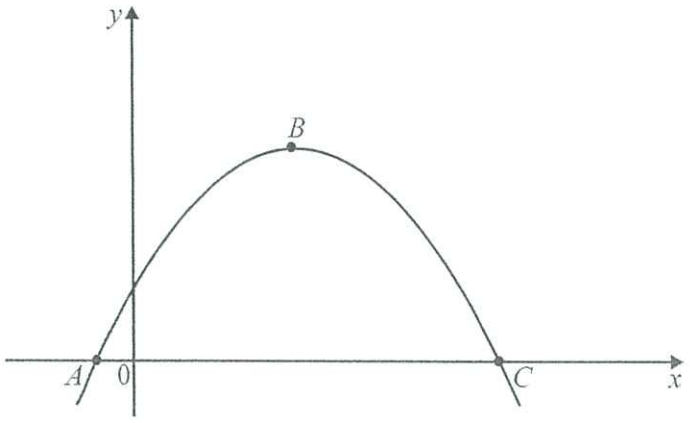
Answers:

(a) $p = -\frac{1}{2}$, $q = 2$.

(b) $C = 3/4$.

(Total 4 marks)

3. The diagram shows the parabola $y = (7 - x)(1 + x)$. The points A and C are the x-intercepts and the point B is the maximum point.



Find the coordinates of A, B and C.

Working:

$y = (7-x)(1+x)$

$x = -1, 7$.

Point B.

x-coord: $\frac{-1+7}{2} = 3$.

y-coord $y = (7-x)(1+x)$

$= (7-3)(1+3) = 16$

Answer:

$A = (-1, 0)$, $B = (3, 16)$

$C = (7, 0)$.

(Total 4 marks)

4. The quadratic equation $4x^2 + 4kx + 9 = 0$, $k > 0$ has exactly one solution for x.
Find the value of k.

Working: $\Delta = 0$ (one solution).

$\Delta = b^2 - 4ac$

$0 = (4k)^2 - 4(4)(9)$

$0 = 16k^2 - 144$ $\therefore k = \pm 3$

$16k^2 = 144$

$k^2 = 9$

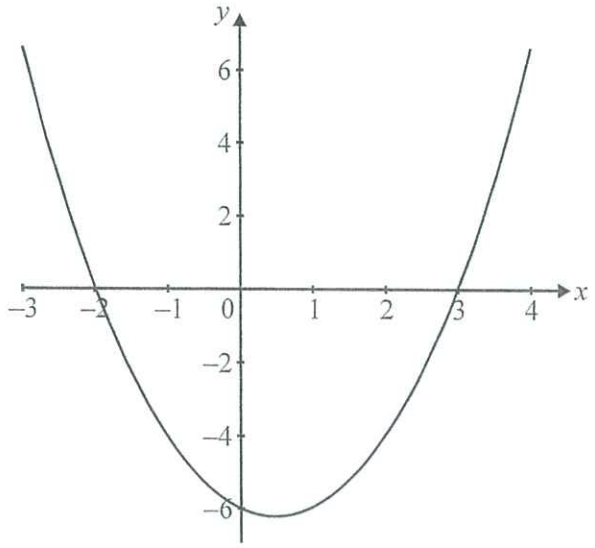
since $k > 0$ $\therefore k = 3$

Answer:

$k = 3$

(Total 4 marks)

5. The diagram shows part of the graph with equation $y = x^2 + px + q$. The graph cuts the x-axis at -2 and 3.



Find the value of

(a) p;

(b) q.

Working: roots @ -2, 3

$y = (x+2)(x-3)$

$y = x^2 - 3x + 2x - 6$

$y = x^2 - x - 6$

Answers:

(a) $p = -1$

(b) $q = -6$

(Total 4 marks)

6. Consider the function $f(x) = 2x^2 - 8x + 5$.

(a) Express $f(x)$ in the form $a(x-p)^2 + q$, where $a, p, q \in \mathbb{Z}$.

(b) Find the minimum value of $f(x)$.

Working:

$f(x) = 2(x^2 - 4x + 4 - 4) + 5$

$= 2(x^2 - 4x + 4) + 5 - 8$

$= 2(x-2)^2 - 3$

(min)

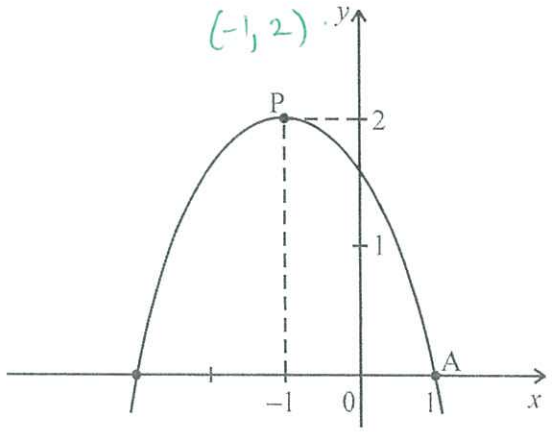
Answers:

(a) $a = 2, p = 2, q = -3 \therefore f(x) = 2(x-2)^2 - 3$

(b) minimum value $= -3$.

(Total 6 marks)

7. The diagram shows part of the graph of $y = a(x - h)^2 + k$. The graph has its vertex at P, and passes through the point A with coordinates (1, 0).



(a) Write down the value of

(i) h;

(ii) k.

(b) Calculate the value of a.

Working:

$y = a(x-h)^2 + k$

$y = a(x+1)^2 + 2$

$0 = a(1+1)^2 + 2$

$-2 = 4a$

$a = -\frac{1}{2}$

Answers:

(a) (i) $h = -1$

(ii) $k = 2$

(b) $a = -\frac{1}{2}$

(Total 6 marks)

8. The equation $kx^2 + 3x + 1 = 0$ has exactly one solution. Find the value of k.

Working:

$\Delta = 0$

$b^2 - 4ac$

$0 = (3)^2 - 4(k)(1)$

$0 = 9 - 4k$

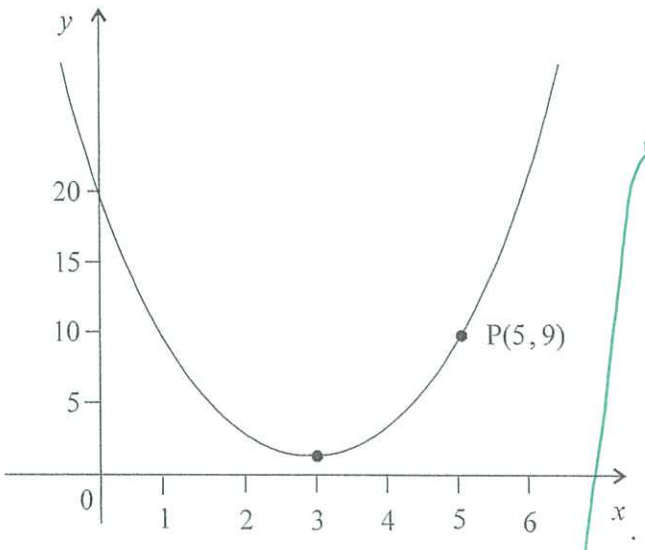
$-9 = -4k$

$k = \frac{9}{4}$

Answer: $k = 9/4$ .....

(Total 6 marks)

9. The diagram shows part of the graph of the curve $y = a(x - h)^2 + k$, where $a, h, k \in \mathbb{Z}$.



$y = a(x-3)^2 + 1$.
point (5, 9)
$\therefore 9 = a(5-3)^2 + 1$.
$9 = a(2)^2 + 1$.
$8 = 4a$
$a = 2$

(a) The vertex is at the point (3, 1). Write down the value of h and of k.

$h = 3,\ k = 1$

(2)

(b) The point P (5, 9) is on the graph. Show that $a = 2$.

(3)

(c) Hence show that the equation of the curve can be written as

$$y = 2x^2 - 12x + 19.$$

(1)

(Total 6 marks)

10. The equation $x^2 - 2kx + 1 = 0$ has two distinct real roots. Find the set of all possible values of k.

Working:

$b^2 - 4ac > 0$

$(-2k)^2 - 4(1)(1) > 0$

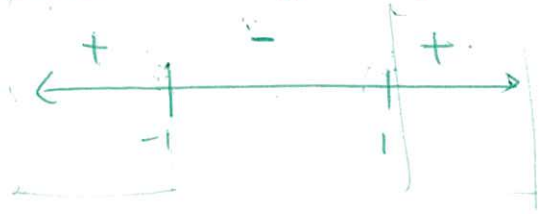
$4k^2 - 4 > 0$

Solve $4k^2 = 4$

$k^2 = 1$

$k = \pm 1$

check w/ sign diagram



Answer:

$\therefore k < -1,\ k > 1$

(Total 6 marks)