1. Find the **exact** value of x in each of the following equations.

(a) $5^{x+1} = 625$

$625 = 5^4$

$5^{x+1} = 5^4$

$x+1 = 4$

$x = 3$ //

(Total 3 marks)

2. The functions $f(x)$ and $g(x)$ are defined by $f(x) = e^x$ and $g(x) = \ln(1+2x)$.

(a) Write down $f^{-1}(x)$.

(b) (i) Find $(f \circ g)(x)$.

(ii) Find $(f \circ g)^{-1}(x)$.

ln = $\log_e$ inverse of e^x = ln

a) $f(x) = e^x$

$y = e^x$

$x = e^y$ using definition of logs

$\log_e x = y$

$\therefore y = \ln x$

$\therefore f^{-1}(x) = \ln x$

b) i) $f(g(x))$ * since $e^{\ln 1} = 1$

$= e^{\ln(1+2x)}$

$= 1+2x$

(Total 6 marks)

b) ii) $(f \circ g)^{-1}(x)$

$y = 1+2x$

$x = 1+2y$

$2y = x-1$

$y = \frac{x-1}{2}$

$\therefore (f \circ g)^{-1}(x) = \frac{x-1}{2}$

3. The population of a city at the end of 1972 was 250 000. The population increases by 1.3% per year.

(a) Write down the population at the end of 1973.

(b) Find the population at the end of 2002.

1.3% = 0.013.

$P = a(1+r)^n$

a) $P = 250000(1+0.013)^n$

$= 253250$

b) end of 2002 $\therefore n = 30$

$P = 25000(1+0.013)^{30}$

$= 368318.3611$

$= 360000$ round to 3 sig figs

(Total 6 marks)

4. Michele invested 1500 francs at an annual rate of interest of 5.25 percent, compounded annually. $FV = P(1+r)^n$

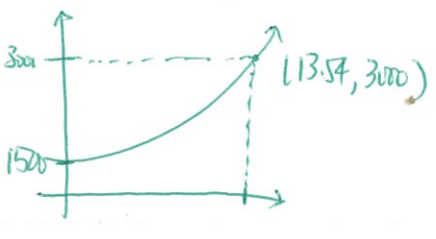
(a) Find the value of Michele's investment after 3 years. Give your answer to the nearest franc. $FV = 1500(1+0.0525)^3 = 1749,,$ **(3)**

(b) How many complete years will it take for Michele's initial investment to double in value? $3000 = 1500(1+0.0525)^t$ $\therefore$ It would take 14 yrs **(3)**

(c) What should the interest rate be if Michele's initial investment were to double in value in 10 years? **(4)**

(Total 10 marks)

using GDC.



c)

$3000 = 1500(1+r)^{10}$

$\sqrt[10]{\frac{3000}{1500}} = (1+r)^{10}$

$\sqrt[10]{2} = (1+r)$

$1.071773463 = (1+r)$

$r = 0.071773463$

$\therefore$ 7.18%

5. Initially a tank contains 10 000 litres of liquid. At the time $t = 0$ minutes a tap is opened, and liquid then flows out of the tank. The volume of liquid, V litres, which remains in the tank after t minutes is given by

$$V = 10\,000\,(0.933^t).$$

(a) Find the value of V after 5 minutes. **(1)**

$V = 10000(0.933^5) = 7069.81 = 7070$ ml.

(b) Find how long, to the nearest second, it takes for half of the initial amount of liquid to flow out of the tank. **(3)**

$5000 = 10000(0.933)^t$

$0.5 = (0.933)^t$

∴ 10 seconds

(c) The tank is regarded as effectively empty when 95% of the liquid has flowed out. Show that it takes almost three-quarters of an hour for this to happen. **(3)**

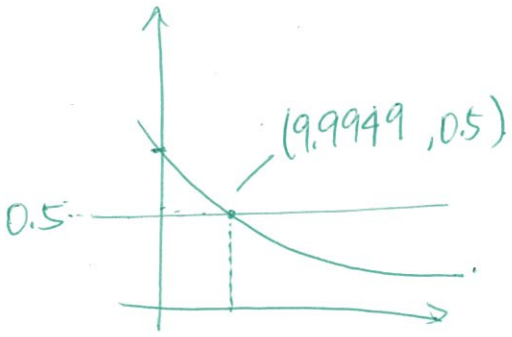
(d) (i) Find the value of $10\,000 - V$ when $t = 0.001$ minutes.

(ii) Hence or otherwise, estimate the initial flow rate of the liquid. Give your answer in litres per minute, correct to two significant figures. **(3)**

(Total 10 marks)

b) $0.5 = (0.933)^t$

using GDC



∴ 10 seconds

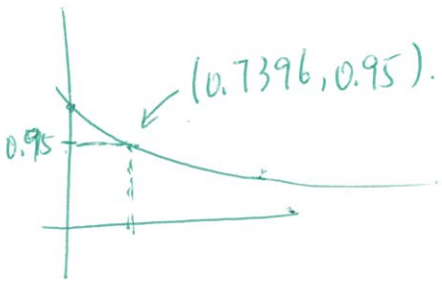
c)

0.95×10000

$= 9500$

$\frac{9500}{10000} = \frac{10000(0.933)^t}{10000}$

$0.95 = 0.933^t$



0.73 → 0.74 is almost 3 quarters of an hour

∴ this is true.