## 3. Scalar (dot) product

The most common use for the scalar product is to find the angle between two vectors. In particular, it is used to find out if two vectors are at right-angles to each other. If two vectors are at right-angles to each other then they are called perpendicular or orthogonal.

The formula for the scalar product is:

$$a \cdot b = |a||b|\cos\theta$$

where $\theta$ is the angle between the vectors.

Also $\boldsymbol{a} \cdot \boldsymbol{b}$ is used as shorthand for $\sum_{i=1}^{n} a_i b_i$

Where $a_i$ and $b_i$ are the components of a and b

$$a \cdot b = a_1 b_1 + a_2 b_2 + a_3 b_3$$

### Examples

1. Given $\underline{a} = (3,4)$, $\underline{b} = (1,2)$ find $\underline{a} \cdot \underline{b}$.

$$\underline{a} \cdot \underline{b} = 3 \times 1 + 4 \times 2 = 3 + 8 = 11$$

2. Given $\underline{a} = (2,3,1)$, $\underline{b} = (1,-2,4)$ find $\underline{a} \cdot \underline{b}$. Comment on your result.

$$\begin{aligned} a \cdot b &= (2 \times 1) + (3 \times -2) + (1 \times 4) \\ &= 2 - 6 + 4 = 0 \end{aligned}$$

As $\underline{a} \cdot \underline{b} = 0$, this means that the vectors are **perpendicular** to each other.

3. Find the angle between $\underline{a} = (5,2)$ and $\underline{b} = (2,-3)$.

$$\underline{a} \cdot \underline{b} = 5 \times 2 + 2 \times -3 = 10 - 6 = 4$$

$$|\underline{a}| = \sqrt{5^2 + 2^2} = \sqrt{29} \qquad |\underline{b}| = \sqrt{2^2 + (-3^2)} = \sqrt{13}$$

$$a \cdot b = |a||b|\cos\theta$$

$$4 = \sqrt{29}\sqrt{13}\cos\theta$$

$$\cos\theta = \frac{4}{\sqrt{29}\sqrt{13}}$$

$$\cos\theta = 0.206$$

$\theta = 78.11$ The angle between $\underline{a}$ and $\underline{b}$ is approximately 78.11 degrees.

## Exercise 3

1 Find the value of $\underline{a} \cdot \underline{b}$ for the following pairs of vectors:

a) $\underline{a} = (2,5), \quad \underline{b} = (-4,6)$

b) $\underline{a} = (1,2,4), \quad \underline{b} = (2,-9,3)$

c) $\underline{a} = (-3,2,7), \quad \underline{b} = (1,4,9)$

2 Use the scalar product to find the angle between the vectors, rounding your answers to 2d.p.

a) u = (1, 2, 3) v = (0, -2, -1)

b) $\underline{a} = (3,2,4), \quad \underline{b} = (1,-6,3)$

c) **r** = $\begin{pmatrix} -3 \\ 1 \\ 5 \end{pmatrix}$ **s** = $\begin{pmatrix} -1 \\ -5 \\ 3 \end{pmatrix}$

d) **m** = $\begin{pmatrix} 0 \\ 3 \\ 3 \end{pmatrix}$ **n** = $\begin{pmatrix} 3 \\ 0 \\ 3 \end{pmatrix}$

e) $\boldsymbol{p} = -2\boldsymbol{i} + 3\boldsymbol{j} + \boldsymbol{k}$ $\quad \boldsymbol{e} = 4\boldsymbol{i} + 7\boldsymbol{j} + 3\boldsymbol{k}$

f) $\boldsymbol{e} = 2\boldsymbol{i} - 2\boldsymbol{j} + 6\boldsymbol{k}$ $\quad \boldsymbol{f} = 5\boldsymbol{i} + 9\boldsymbol{j} + 13\boldsymbol{k}$

3. Find the value of $x$ if the vectors $\boldsymbol{u} = \begin{pmatrix} -2 \\ -5 \\ 1 \end{pmatrix}$ and $\boldsymbol{v} = \begin{pmatrix} x \\ -2 \\ 6 \end{pmatrix}$ are perpendicular.

4. Find the value of $y$ if the vectors $\boldsymbol{u} = \begin{pmatrix} y \\ -1 \\ 1 \end{pmatrix}$ and $\boldsymbol{v} = \begin{pmatrix} 2 \\ -3 \\ -1 \end{pmatrix}$ are perpendicular.

5. Evaluate $\overrightarrow{AB} \cdot \overrightarrow{OC}$ for the points A(3, 2, 1) B(4, 5, -2) C(4, 2, 3).