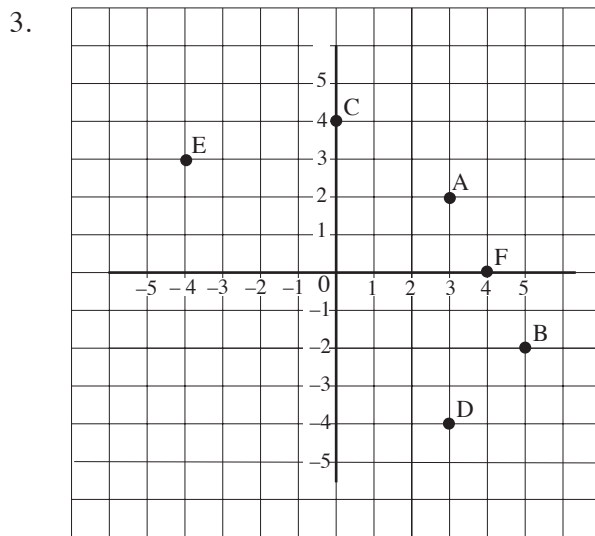# 19 | Vectors

19.1 | Vectors and Scalars

1.   Vectors: (b), (d), (e)   scalars: (a), (c), (f)

2.   (a) $\begin{pmatrix} 6 \\ 0 \end{pmatrix}$ (b) $\begin{pmatrix} 6 \\ -3 \end{pmatrix}$ (c) $\begin{pmatrix} -3 \\ 2 \end{pmatrix}$ (d) $\begin{pmatrix} -6 \\ -3 \end{pmatrix}$

      (e) $\begin{pmatrix} 6 \\ 3 \end{pmatrix}$ (f) $\begin{pmatrix} 3 \\ -5 \end{pmatrix}$ (g) $\begin{pmatrix} -3 \\ -2 \end{pmatrix}$ (h) $\begin{pmatrix} 3 \\ 2 \end{pmatrix}$

3.

$$\text{EF} = \begin{pmatrix} 8 \\ -3 \end{pmatrix}$$

4.   (a) $\begin{pmatrix} 7 \\ 2 \end{pmatrix}$ (b) $\begin{pmatrix} 3 \\ -1 \end{pmatrix}$ (c) $\begin{pmatrix} 4 \\ 11 \end{pmatrix}$ (d) $\begin{pmatrix} 1 \\ 12 \end{pmatrix}$

      (e) $\begin{pmatrix} -1 \\ -12 \end{pmatrix}$ (f) $\begin{pmatrix} -4 \\ -3 \end{pmatrix}$ (g) $\begin{pmatrix} 12 \\ 21 \end{pmatrix}$ (h) $\begin{pmatrix} -6 \\ 10 \end{pmatrix}$
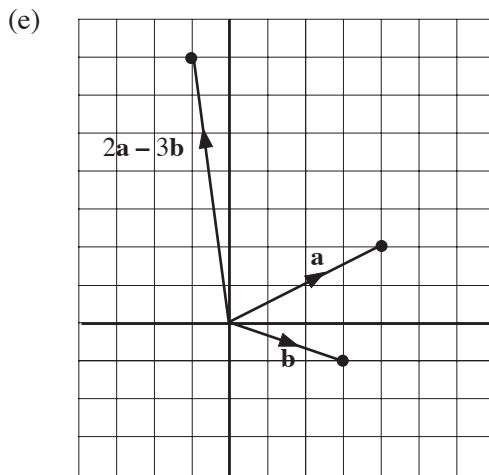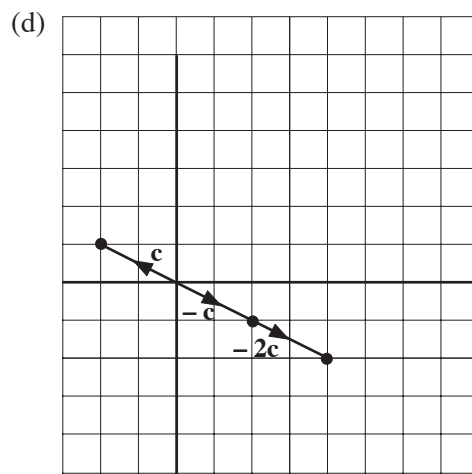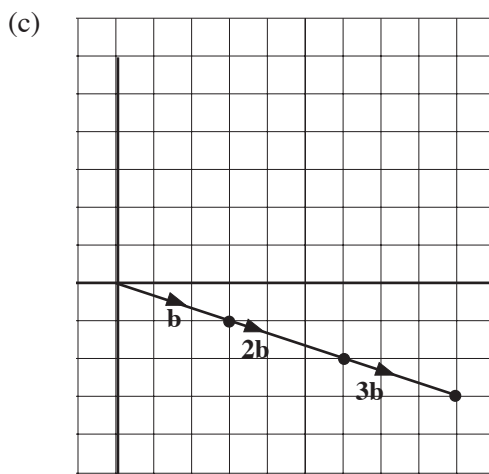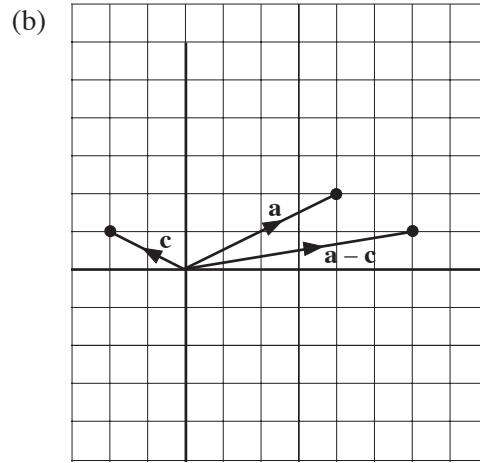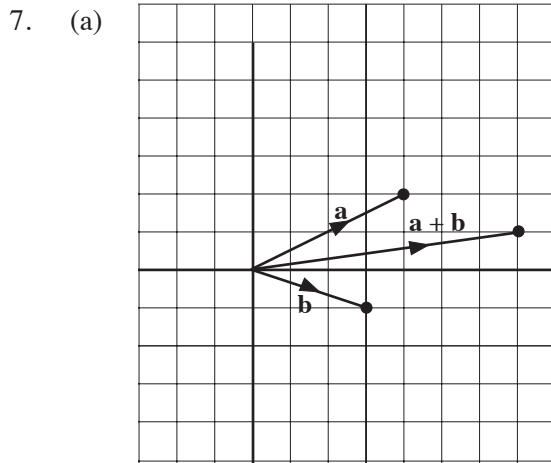
      (i) $\begin{pmatrix} 0 \\ 16 \end{pmatrix}$ (j) $\begin{pmatrix} 17 \\ -1 \end{pmatrix}$ (k) $\begin{pmatrix} -12 \\ -1 \end{pmatrix}$ (l) $\begin{pmatrix} 12 \\ -28 \end{pmatrix}$

5.   (a) $\begin{pmatrix} 10 \\ 1 \end{pmatrix}$ (b) $\begin{pmatrix} 2 \\ -8 \end{pmatrix}$ (c) $\begin{pmatrix} 0 \\ 9 \end{pmatrix}$ (d) $\begin{pmatrix} -4 \\ -14 \end{pmatrix}$

      (e) $\begin{pmatrix} -10 \\ -14 \end{pmatrix}$ (f) $\begin{pmatrix} 24 \\ 21 \end{pmatrix}$

6.   (a) $\begin{pmatrix} 1 \\ -3 \end{pmatrix}$ (b) $\begin{pmatrix} 6 \\ -2 \end{pmatrix}$ (c) $\begin{pmatrix} 1 \\ -1 \end{pmatrix}$ (d) $\begin{pmatrix} \frac{1}{2} \\ -\frac{3}{2} \end{pmatrix}$ (e) $\begin{pmatrix} -1 \\ -3 \end{pmatrix}$

      (f) $\begin{pmatrix} 4 \\ 7 \end{pmatrix}$ (g) $\begin{pmatrix} 3 \\ -\frac{11}{2} \end{pmatrix}$ (h) $\begin{pmatrix} -7 \\ \frac{13}{2} \end{pmatrix}$ (i) $\begin{pmatrix} -5 \\ 3 \end{pmatrix}$

7. (a)

(b)

(c)

(d)

(e)

19.2 Applications of Vectors
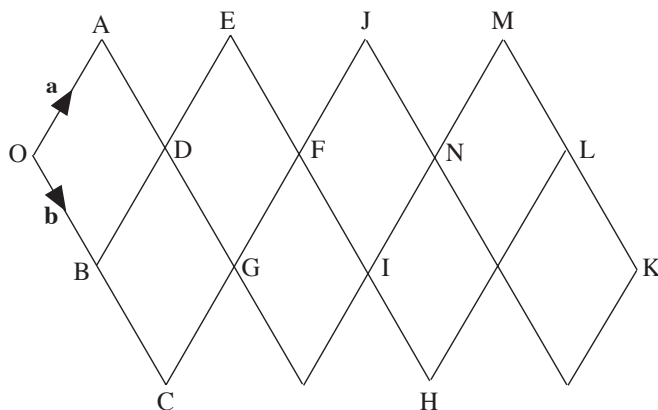
1. (a) 3.4 m/s ,   63°   (b) $3\frac{1}{3}$ s   (c) 5 m

2. 2.5 m/s,   1.875 m/s

3. (a) 34 m/s   (b) 081° , 62.5 m/s

4. 37° to bank;   20 s

5. (a) bearing 343° ,   speed 209 km/h   (b) bearing 017.5°

6.   408 seconds

7.   (b) $P = 577$ N, $Q = 289$ N

8. $R = 580$ N, $F = 155$ N

9.   0.64 m/s

10. 93.75 m

11. (a) (i) 0.85 m/s   (ii) 28°   (b) (i) 8 m   (ii) 20 s

19.3 Vectors and Geometry

1.   (a) **4a**   (b) **a**   (c) **b**   (d) **−a + 2b**

     (e) **b**   (f) **3a + 2b**   (g) **3b**   (h) **3b**

     (i) **a − b**   (j) **2a − 2b**   (k) **−a − 2b**   (l) **−3a − 2b**

     (m) **−3a + 3b** (n) **−2a**   (o) **−4a + b**   (p) **−3a − 3b**

2.

3.   (a) (i) **c**   (ii) **a**   (iii) **− a**   (iv) **− a + c**   (v) **a + c**   (vi) **a − c**

     (b) (i) $\frac{1}{2}$**c**   (ii) **a** $+ \frac{1}{2}$**c**   (iii) $\frac{1}{2}$**a**   (iv) **c** $+ \frac{1}{2}$**a**   (v) $-\frac{1}{2}$**a** $+ \frac{1}{2}$**c**

4.   (a) $\frac{1}{2}$(**q** + **p**)   (b) $\frac{1}{2}$(**p** + **q**)   (c) M and N are coincident

5.   (a) $\vec{AD} = 6$**i**, $\vec{OD} = 6$**i** $+ 6$**j**

     (b) $\vec{CE} = 4$**j**, $\vec{OE} = 8$**i** $+ 4$**j**

     (c) $\vec{OM} = 7$**i** $+ 5$**j**

6.   (a) $\frac{1}{2}$(**p** + **q**), $\frac{1}{2}$**p** $+ \frac{5}{2}$**q**   (b) **2q**

8.   (a) **2a** + **b** + **c**   (b) **a** $+ \frac{1}{2}$**b** $+ \frac{1}{2}$**c**   (c) **− a**   (d) **a** $+ \frac{1}{2}$**b** $- \frac{1}{2}$**c**

     (e) $\frac{1}{2}$(**c** − **b**)

10. (a) $\dfrac{1}{3}\mathbf{b} - \mathbf{d}$ (b) $\mathbf{b} + \mathbf{d}$ (c) $\alpha = \beta = 3$

11. $\overrightarrow{AQ} = \dfrac{1}{3}(\mathbf{a} + \mathbf{b})$

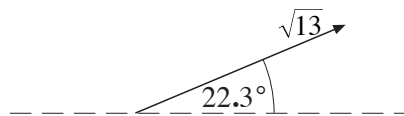12. (a) $\overrightarrow{AC} = 2\mathbf{p} + 8\mathbf{q}$

13. (a) (i) $-\mathbf{a} + \dfrac{1}{2}\mathbf{c}$ (ii) $\mathbf{c} - 2\mathbf{a}$ (b) $1 : 2$
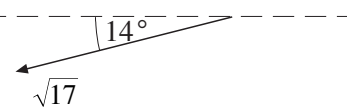
19.4   Further Work with Vectors

1. (a) $\begin{pmatrix} 40\cos 20° \\ 40\sin 20° \end{pmatrix}$ (b) $\begin{pmatrix} 30\cos 80° \\ 30\sin 80° \end{pmatrix}$ (c) $\begin{pmatrix} 8\cos 30° \\ 8\sin 30° \end{pmatrix}$

   (d) $\begin{pmatrix} 7\cos 20° \\ -7\sin 20° \end{pmatrix}$ (e) $\begin{pmatrix} -12\cos 40° \\ 12\sin 40° \end{pmatrix}$ (f) $\begin{pmatrix} -10\cos 38° \\ -10\sin 38° \end{pmatrix}$
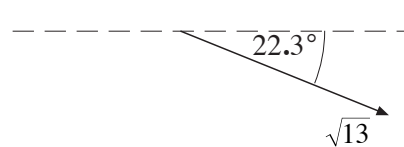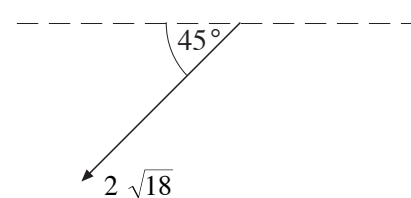
2. (a)
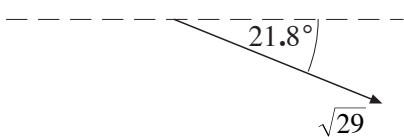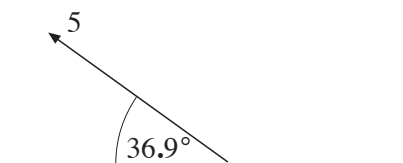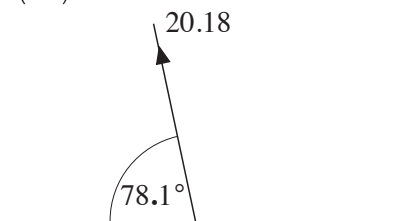
   (b)

   (c)

   (d)

   (e)

   (f)

3. $\begin{pmatrix} 5 \\ -3 \end{pmatrix}$, $\sqrt{34}$

4.

5. $\begin{pmatrix} 3 \\ -1.2 \end{pmatrix}$, 3.23 m/s at 111.8°

6. 242 m/s, 221.4°

7. 226.4 m/s at 169.2°

8. $F \approx 7.21$ N, $\theta \approx 43.9°$

9. $\theta \approx 23.6°$, 80.9 N

10. $\theta \approx 59.3°$, 386.5 N

11. (a) (i) $2\cos a$ (ii) $1 - 2\sin\alpha$ (b) 30°

19.5 Commutative and Associative Properties

1. $\mathbf{a} - \mathbf{b} = \begin{pmatrix} -1 \\ 3 \end{pmatrix}$, $\mathbf{b} - \mathbf{a} = \begin{pmatrix} 1 \\ -3 \end{pmatrix}$, $\mathbf{a} - \mathbf{b} \neq \mathbf{b} - \mathbf{a}$

5. $(\mathbf{a} + \mathbf{b}) + \mathbf{c} = \begin{pmatrix} 6 \\ 1 \end{pmatrix} = \mathbf{a} + (\mathbf{b} + \mathbf{c})$