**1** **a** $\log_{10} 1000 = 3$ **b** $\log_3 81 = 4$ **c** $\log_2 256 = 8$ **d** $\log_7 1 = 0$

**e** $\log_3 \frac{1}{27} = -3$ **f** $\log_{32} \frac{1}{2} = -\frac{1}{5}$ **g** $\log_{19} 19 = 1$ **h** $\log_{36} 216 = \frac{3}{2}$

**2** **a** $5^3 = 125$ **b** $2^4 = 16$ **c** $10^5 = 100\,000$ **d** $23^0 = 1$

**e** $9^{\frac{1}{2}} = 3$ **f** $10^{-2} = 0.01$ **g** $2^{-3} = \frac{1}{8}$ **h** $6^1 = 6$

**3** **a** $= \log_7 7^2 = 2$ **b** $= \log_4 4^3 = 3$ **c** $= \log_2 2^7 = 7$ **d** $= \log_3 3^3 = 3$

**e** $= \log_5 5^4 = 4$ **f** $= \log_8 8^1 = 1$ **g** $= \log_7 7^0 = 0$ **h** $= \log_{15} 15^{-1} = -1$

**i** $= \log_3 3^{-2} = -2$ **j** $= \lg 10^{-3} = -3$ **k** $= \log_{16} 16^{\frac{1}{4}} = \frac{1}{4}$ **l** $= \log_4 4^{\frac{3}{2}} = \frac{3}{2}$

**m** $= \log_9 9^{\frac{5}{2}} = \frac{5}{2}$ **n** $= \log_{100} 100^{-\frac{3}{2}} = -\frac{3}{2}$ **o** $= \log_{25} 25^{\frac{3}{2}} = \frac{3}{2}$ **p** $= \log_{27} 27^{-\frac{2}{3}} = -\frac{2}{3}$

**4** **a** $5^x = 25$, $x = 2$ **b** $2^6 = x$, $x = 64$ **c** $x^3 = 64$, $x = 4$ **d** $10^{-3} = x$, $x = \frac{1}{1000}$

**e** $x^{\frac{2}{3}} = 16$, $x = 64$ **f** $5^x = 1$, $x = 0$ **g** $x^1 = 9$, $x = 9$ **h** $10^x = 10^{12}$, $x = 12$

**i** $\log_x 7 = \frac{1}{2}$, $x^{\frac{1}{2}} = 7$, $x = 49$ **j** $4^{1.5} = x$, $x = 8$ **k** $x^{-\frac{1}{3}} = 0.1$, $x = 1000$ **l** $\log_8 x = -\frac{1}{3}$, $8^{-\frac{1}{3}} = x$, $x = \frac{1}{2}$

**5** **a** $= \log_a (4 \times 7) = \log_a 28$ **b** $= \log_a (10 \div 5) = \log_a 2$ **c** $= \log_a 6^2 = \log_a 36$

**d** $= \log_a (9 \div \frac{1}{3}) = \log_a 27$

**e** $= \log_a 25^{\frac{1}{2}} + \log_a 3^2 = \log_a 5 + \log_a 9 = \log_a (5 \times 9) = \log_a 45$

**f** $= \log_a 48 - \log_a 2^3 - \log_a 9^{\frac{1}{2}} = \log_a 48 - \log_a 8 - \log_a 3 = \log_a [48 \div (8 \times 3)] = \log_a 2$

**6** **a** $= 5 \log_q x$ **b** $= \frac{15}{2} \log_q x$ **c** $= \log_q x^{-1} = -\log_q x$ **d** $= \log_q x^{\frac{1}{3}} = \frac{1}{3} \log_q x$

**e** $= 4 \log_q x^{-\frac{1}{2}} = -2 \log_q x$ **f** $= 2 \log_q x + 5 \log_q x = 7 \log_q x$ **g** $= \log_q x^{-2} + \log_q x^{-3} = -2 \log_q x - 3 \log_q x = -5 \log_q x$ **h** $= 6 \log_q x - 2 \log_q x = 4 \log_q x$

**7**

**a** $= \lg (5 \times 4)$
$= \lg 20$

**b** $= \lg (12 \div 6)$
$= \lg 2$

**c** $= \lg 2^3$
$= \lg 8$

**d** $= \lg 3^4 - \lg 9$
$= \lg 81 - \lg 9$
$= \lg (81 \div 9)$
$= \lg 9$

**e** $= \lg 16^{\frac{1}{2}} - \lg 32^{\frac{1}{5}}$
$= \lg 4 - \lg 2$
$= \lg (4 \div 2)$
$= \lg 2$

**f** $= \lg 10 + \lg 11$
$= \lg (10 \times 11)$
$= \lg 110$

**g** $= \lg \frac{1}{50} + \lg 10^2$
$= \lg \frac{1}{50} + \lg 100$
$= \lg (\frac{1}{50} \times 100)$
$= \lg 2$

**h** $= \lg 10^3 - \lg 40$
$= \lg 1000 - \lg 40$
$= \lg (1000 \div 40)$
$= \lg 25$

**8**

**a** $= \log_3 (54 \div 2)$
$= \log_3 27$
$= \log_3 3^3$
$= 3$

**b** $= \log_5 (20 \times 1.25)$
$= \log_5 25$
$= \log_5 5^2$
$= 2$

**c** $= \log_2 2^4 + \log_3 3^3$
$= 4 + 3$
$= 7$

**d** $= \log_6 (24 \times 9)$
$= \log_6 216$
$= \log_6 6^3$
$= 3$

**e** $= \log_3 (12 \div 4)$
$= \log_3 3$
$= 1$

**f** $= \log_4 (18 \div 9)$
$= \log_4 2$
$= \log_4 4^{\frac{1}{2}}$
$= \frac{1}{2}$

**g** $= \log_9 (4 \times 0.25)$
$= \log_9 1$
$= 0$

**h** $= \lg 2^2 + \lg 25$
$= \lg 4 + \lg 25$
$= \lg (4 \times 25)$
$= \lg 100$
$= \lg 10^2$
$= 2$

**i** $= \log_3 8^{\frac{1}{3}} - \log_3 18$
$= \log_3 2 - \log_3 18$
$= \log_3 (2 \div 18)$
$= \log_3 \frac{1}{9}$
$= \log_3 3^{-2}$
$= -2$

**j** $= \log_4 64^{\frac{1}{3}} + (2 \times \log_5 5^2)$
$= \log_4 4 + (2 \times 2)$
$= 1 + 4$
$= 5$

**k** $= \frac{1}{2} \log_5 \frac{25}{16} + \log_5 10^2$
$= \log_5 (\frac{25}{16})^{\frac{1}{2}} + \log_5 100$
$= \log_5 \frac{5}{4} + \log_5 100$
$= \log_5 (\frac{5}{4} \times 100)$
$= \log_5 125$
$= \log_5 5^3$
$= 3$

**l** $= \log_3 5 - \log_3 6^2 - \log_3 \frac{15}{4}$
$= \log_3 [5 \div (36 \times \frac{15}{4})]$
$= \log_3 \frac{1}{27}$
$= \log_3 3^{-3}$
$= -3$

**1** **a** $= \log_{10} a + \log_{10} b$

**b** $= \log_{10} a + \log_{10} b^7$
$= \log_{10} a + 7\log_{10} b$

**c** $= \log_{10} a^3 - \log_{10} b$
$= 3\log_{10} a - \log_{10} b$

**d** $= \log_{10} a + \log_{10} b^{\frac{1}{2}}$
$= \log_{10} a + \frac{1}{2}\log_{10} b$

**e** $= 2\log_{10} ab$
$= 2\log_{10} a + 2\log_{10} b$

**f** $= -\log_{10} ab$
$= -\log_{10} a - \log_{10} b$

**g** $= \log_{10} a^{\frac{3}{2}} + \log_{10} b^{\frac{5}{2}}$
$= \frac{3}{2}\log_{10} a + \frac{5}{2}\log_{10} b$

**h** $= 3(\log_{10} a^2 - \log_{10} b^{\frac{1}{3}})$
$= 6\log_{10} a - \log_{10} b$

**2** **a** $= \log_q 8^2$
$= 2y$

**b** $= \log_q 8^{\frac{1}{3}}$
$= \frac{1}{3}y$

**c** $= \log_q 16 - \log_q q$
$= \log_q 8^{\frac{4}{3}} - 1$
$= \frac{4}{3}y - 1$

**d** $= \log_q 4 + \log_q q^3$
$= \log_q 8^{\frac{2}{3}} + 3$
$= \frac{2}{3}y + 3$

**3** **a** $= \lg(2 \times 3^2)$
$= \lg 2 + 2\lg 3$
$= a + 2b$

**b** $= \lg(2^5 \times 3)$
$= 5\lg 2 + \lg 3$
$= 5a + b$

**c** $= \lg 9 - \lg 16$
$= \lg 3^2 - \lg 2^4$
$= 2\lg 3 - 4\lg 2$
$= 2b - 4a$

**d** $= \lg(2 \times 3) - \lg 2^3$
$= \lg 2 + \lg 3 - 3\lg 2$
$= \lg 3 - 2\lg 2$
$= b - 2a$

**e** $= \frac{1}{2}\lg 6$
$= \frac{1}{2}(\lg 2 + \lg 3)$
$= \frac{1}{2}(a + b)$

**f** $= \frac{3}{2}\lg 2^4 + \frac{1}{2}\lg 3^4$
$= 6\lg 2 + 2\lg 3$
$= 6a + 2b$

**g** $= 4\lg 3 - 3(\lg 2 + \lg 3)$
$= \lg 3 - 3\lg 2$
$= b - 3a$

**h** $= \lg(6 \times 10) + \lg(2 \times 10) - 2$
$= \lg 6 + 1 + \lg 2 + 1 - 2$
$= \lg 2 + \lg 3 + \lg 2$
$= 2a + b$

**4** **a** $= \log_5 10 - \log_5 2$
$= \log_5 5$
$= 1$

**b** $= \log_{12} 16 + \log_{12} 9$
$= \log_{12} 144$
$= 2$

**c** $= \log_4 8$
$= \log_4 4^{\frac{3}{2}}$
$= \frac{3}{2}$

**d** $= \dfrac{\log_7 3^4}{\log_7 3}$
$= \dfrac{4\log_7 3}{\log_7 3}$
$= 4$

**e** $= \log_{27} \dfrac{12^3}{72^2}$
$= \log_{27} \dfrac{12 \times 12 \times 12}{6 \times 12 \times 6 \times 12}$
$= \log_{27} \frac{1}{3} = -\frac{1}{3}$

**f** $= \dfrac{\log_{11} 5^2}{-\log_{11} 5}$
$= \dfrac{2\log_{11} 5}{-\log_{11} 5}$
$= -2$

**5** **a** $x = 3^{1.8}$
$x = 7.22$

**b** $x = 5^{-0.3}$
$x = 0.617$

**c** $x - 3 = 8^{2.1}$
$x = 3 + 8^{2.1}$
$x = 81.8$

**d** $\frac{1}{2}x + 1 = 4^{3.2}$
$x = 2(4^{3.2} - 1)$
$x = 167$

**e** $\log_2 3y = 5.3$
$3y = 2^{5.3}$
$y = \frac{1}{3} \times 2^{5.3}$
$y = 13.1$

**f** $\log_6 (1 - 5t) = -0.6$
$1 - 5t = 6^{-0.6}$
$t = \frac{1}{5}(1 - 6^{-0.6})$
$t = 0.132$

**6** **a** $= \log_2 x^5$

**b** $= \log_2 (x^2 + 4x)$

**c** $= \log_2 x^2 + \log_2 x$
$= \log_2 x^3$

**d** $= \log_2 (x - 2)^3 - \log_2 x^4$
$= \log_2 \dfrac{(x-2)^3}{x^4}$

**e** $= \log_2 \dfrac{x^2 - 1}{x + 1}$
$= \log_2 \dfrac{(x+1)(x-1)}{x+1}$
$= \log_2 (x - 1)$

**f** $= \log_2 x - 2\log_2 x + \frac{2}{3}\log_2 x$
$= -\frac{1}{3}\log_2 x$
$= \log_2 x^{-\frac{1}{3}}$

**7** **a** $\log_3 5x = \log_3 (2x + 3)$

$5x = 2x + 3$
$x = 1$

**b** $\log_9 10x = \frac{3}{2}$

$10x = 9^{\frac{3}{2}} = 27$
$x = 2.7$

**c** $\log_4 \frac{x}{x-1} = \log_4 3 + \log_4 2 = \log_4 6$

$\frac{x}{x-1} = 6$
$x = 6x - 6$
$x = \frac{6}{5}$

**d** $\log_5 \frac{5x}{x+2} = \log_5 \frac{x+6}{x}$

$\frac{5x}{x+2} = \frac{x+6}{x}$
$5x^2 = (x + 2)(x + 6) = x^2 + 8x + 12$
$x^2 - 2x - 3 = 0$
$(x + 1)(x - 3) = 0$
$x = -1, 3$
$\log_5 x$ not real for $x = -1$ $\therefore x = 3$

**e** $\log_6 x^2 = \log_6 5(2x - 5)$

$x^2 = 5(2x - 5)$
$x^2 - 10x + 25 = 0$
$(x - 5)^2 = 0$
$x = 5$

**f** $\log_7 4x - \log_7 \frac{1}{x-6} = 1$

$\log_7 4x(x - 6) = 1$
$4x(x - 6) = 7$
$4x^2 - 24x - 7 = 0$
$x = \frac{24 \pm \sqrt{576+112}}{8} = 3 \pm \frac{1}{2}\sqrt{43}$
$\log_7 4x$ not real for $x = 3 - \frac{1}{2}\sqrt{43}$
$\therefore x = 3 + \frac{1}{2}\sqrt{43}$ [ = 6.28 (3sf)]

**8** **a** $\log_x y = 2 \Rightarrow y = x^2$

sub. $x^3 = 27$
$x = 3$
$\therefore x = 3, y = 9$

**b** $\log_5 x - 2\log_5 y = \log_5 2 \Rightarrow \frac{x}{y^2} = 2$
$\Rightarrow x = 2y^2$

sub. $3y^2 = 12$
$y^2 = 4$
for real $\log_5 y$, $y > 0$ $\therefore y = 2$
$\therefore x = 8, y = 2$

**c** $\log_y 32 = -\frac{5}{2} \Rightarrow y^{-\frac{5}{2}} = 32$
$\Rightarrow y = 32^{-\frac{2}{5}} = \frac{1}{4}$

sub. $\log_2 x = 3 - 2\log_2 \frac{1}{4}$
$\log_2 x = 3 - (-4) = 7$
$x = 2^7 = 128$
$\therefore x = 128, y = \frac{1}{4}$

**d** $\log_y x = \frac{3}{2} \Rightarrow y^{\frac{3}{2}} = x$
$\Rightarrow y^{\frac{1}{2}} = x^{\frac{1}{3}}$

sub. $4x^{\frac{1}{3}} = 20$
$x^{\frac{1}{3}} = 5$
$x = 5^3 = 125$
$\therefore x = 125, y = 25$

**e** $\log_a x + \log_a 3 = \frac{1}{2}\log_a y \Rightarrow 3x = y^{\frac{1}{2}}$
$\Rightarrow y = 9x^2$

sub. $3x + 9x^2 = 20$
$9x^2 + 3x - 20 = 0$
$(3x + 5)(3x - 4) = 0$
for real $\log_a x$, $x > 0$ $\therefore x = \frac{4}{3}$
$\therefore x = \frac{4}{3}, y = 16$

**f** $\log_{10} y + 2\log_{10} x = 3 \Rightarrow x^2y = 10^3$
$\log_2 y - \log_2 x = 3 \Rightarrow \frac{y}{x} = 2^3$
$\Rightarrow y = 8x$

sub. $8x^3 = 1000$
$x^3 = 125$
$x = 5$
$\therefore x = 5, y = 40$

**1** **a** 1.78 **b** 0.778 **c** 2.40 **d** –0.398

**2** **a** $x = \lg 14 = 1.15$

**b** $10^x = 4$
$x = \lg 4 = 0.60$

**c** $3x = \lg 49$
$x = \frac{1}{3} \lg 49 = 0.56$

**d** $x - 4 = \lg 23$
$x = 4 + \lg 23 = 5.36$

**e** $2x + 1 = \lg 130$
$x = \frac{1}{2}(\lg 130 - 1) = 0.56$

**f** $(10^2)^x = 10^{2x} = 5$
$2x = \lg 5$
$x = \frac{1}{2} \lg 5 = 0.35$

**3** let $y = \log_a b \;\Rightarrow\; a^y = b$
$y \log_c a = \log_c b$
$y = \frac{\log_c b}{\log_c a}$
$\therefore\; \log_a b = \frac{\log_c b}{\log_c a}$

**4** **a** $= \frac{\lg 7}{\lg 2} = 2.81$ **b** $= \frac{\lg 172}{\lg 20} = 1.72$ **c** $= \frac{\lg 49}{\lg 5} = 2.42$ **d** $= \frac{\lg 4}{\lg 9} = 0.631$

**5** **a** $x \lg 3 = \lg 12$
$x = \frac{\lg 12}{\lg 3}$
$x = 2.26$

**b** $x \lg 2 = \lg 0.7$
$x = \frac{\lg 0.7}{\lg 2}$
$x = -0.515$

**c** $-y \lg 8 = \lg 3$
$y = -\frac{\lg 3}{\lg 8}$
$y = -0.528$

**d** $\frac{1}{2}x \lg 4 = \lg 0.3$
$x = \frac{2\lg 0.3}{\lg 4}$
$x = -1.74$

**e** $(t + 3) \lg 5 = \lg 24$
$t = \frac{\lg 24}{\lg 5} - 3$
$t = -1.03$

**f** $(4 + x) \lg 3 = \lg 16$
$x = \frac{\lg 16}{\lg 3} - 4$
$x = -1.48$

**g** $(2x + 4)\lg 7 = \lg 12$
$x = \frac{1}{2}(\frac{\lg 12}{\lg 7} - 4)$
$x = -1.36$

**h** $2^{3x+1} = 12.4$
$(3x + 1) \lg 2 = \lg 12.4$
$x = \frac{1}{3}(\frac{\lg 12.4}{\lg 2} - 1)$
$x = 0.877$

**i** $(2 - 3x) \lg 4 = \lg 32.7$
$x = \frac{1}{3}(2 - \frac{\lg 32.7}{\lg 4})$
$x = -0.172$

**j** $x \lg 5 = (x - 1) \lg 6$
$x (\lg 6 - \lg 5) = \lg 6$
$x = \frac{\lg 6}{\lg 6 - \lg 5} = 9.83$

**k** $(y + 2) \lg 7 = (y + 1) \lg 9$
$y (\lg 9 - \lg 7) = 2 \lg 7 - \lg 9$
$y = \frac{2\lg 7 - \lg 9}{\lg 9 - \lg 7} = 6.74$

**l** $(5 - x) \lg 4 = (2x - 1) \lg 11$
$x (2 \lg 11 + \lg 4) = 5 \lg 4 + \lg 11$
$x = \frac{5\lg 4 + \lg 11}{2\lg 11 + \lg 4} = 1.51$

**m** $(\frac{1}{2}x + 3) \lg 4 = (1 - 2x) \lg 5$
$x (\frac{1}{2} \lg 4 + 2 \lg 5) = \lg 5 - 3 \lg 4$
$x = \frac{\lg 5 - 3\lg 4}{\frac{1}{2}\lg 4 + 2\lg 5} = -0.652$

**n** $(3y - 2) \lg 2 = (2y + 5) \lg 3$
$y (3 \lg 2 - 2 \lg 3) = 5 \lg 3 + 2 \lg 2$
$y = \frac{5\lg 3 + 2\lg 2}{3\lg 2 - 2\lg 3} = -58.4$

**o** $7^{2x+4} = 11^{3x-4}$
$(2x + 4) \lg 7 = (3x - 4) \lg 11$
$x (3 \lg 11 - 2 \lg 7) = 4 \lg 7 + 4 \lg 11$
$x = \frac{4\lg 7 + 4\lg 11}{3\lg 11 - 2\lg 7} = 5.26$

**p** $3^{x+1} = 2^{4+x}$
$(x + 1) \lg 3 = (4 + x) \lg 2$
$x (\lg 3 - \lg 2) = 4 \lg 2 - \lg 3$
$x = \frac{4\lg 2 - \lg 3}{\lg 3 - \lg 2} = 4.13$

**6**

**a** $(2^x + 3)(2^x - 2) = 0$
$2^x = -3$ [no sols], 2
$x = 1$

**b** $(3^x - 1)(3^x - 4) = 0$
$3^x = 1, 4$
$x = 0, \frac{\lg 4}{\lg 3} = 0, 1.26$

**c** $5^{2x} - 8(5^x) + 12 = 0$
$(5^x - 2)(5^x - 6) = 0$
$5^x = 2, 6$
$x = \frac{\lg 2}{\lg 5}, \frac{\lg 6}{\lg 5} = 0.43, 1.11$

**d** $2(4^{2x}) - 7(4^x) + 3 = 0$
$(2(4^x) - 1)(4^x - 3) = 0$
$4^x = \frac{1}{2}, 3$
$x = -\frac{1}{2}, \frac{\lg 3}{\lg 4} = -\frac{1}{2}, 0.79$

**e** $2(2^{2y}) + 7(2^y) - 15 = 0$
$(2(2^y) - 3)(2^y + 5) = 0$
$2^y = -5$ [no sols], $\frac{3}{2}$
$y = \frac{\lg \frac{3}{2}}{\lg 2} = 0.58$

**f** $3(3^{2x}) - 17(3^x) + 10 = 0$
$(3(3^x) - 2)(3^x - 5) = 0$
$3^x = \frac{2}{3}, 5$
$x = \frac{\lg \frac{2}{3}}{\lg 3}, \frac{\lg 5}{\lg 3} = -0.37, 1.46$

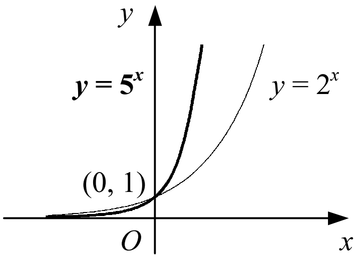
**g** $5^{2t} + 5(5^t) - 24 = 0$
$(5^t + 8)(5^t - 3) = 0$
$5^t = -8$ [no sols], 3
$t = \frac{\lg 3}{\lg 5} = 0.68$

**h** $3(3^{2x}) - 18(3^x) + 15 = 0$
$3(3^x - 1)(3^x - 5) = 0$
$3^x = 1, 5$
$x = 0, \frac{\lg 5}{\lg 3} = 0, 1.46$

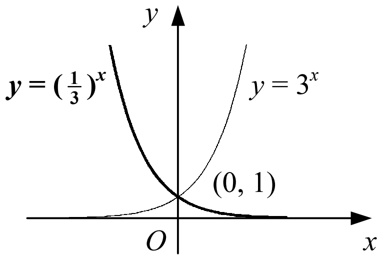
**i** $3(4^{2x}) - 16(4^x) + 5 = 0$
$(3(4^x) - 1)(4^x - 5) = 0$
$4^x = \frac{1}{3}, 5$
$x = \frac{\lg \frac{1}{3}}{\lg 4}, \frac{\lg 5}{\lg 4} = -0.79, 1.16$

**7**

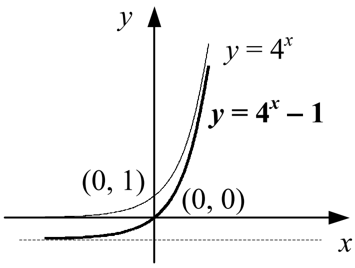
**a**



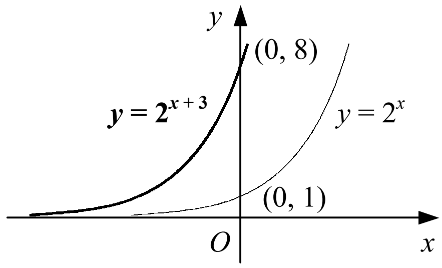

**b**




**c**

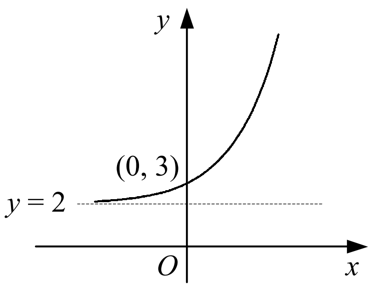



**d**




**8**

**a**




**b** $(3, 29) \Rightarrow 29 = 2 + a^3$
$a^3 = 27$
$a = 3$

**9**

$x = 0 \Rightarrow y = -4$
$y = 0 \Rightarrow 2^x = 5$
$x = \frac{\lg 5}{\lg 2}$
$AB^2 = 4^2 + (\frac{\lg 5}{\lg 2})^2 = 21.391$
$AB = 4.63$

**1** **a** $= \log_{10} \frac{3}{2}$

$= \log_{10} 3 - \log_{10} 2$

$= b - a$

**b** $= \log_{10} (2^3 \times 3)$

$= 3 \log_{10} 2 + \log_{10} 3$

$= 3a + b$

**c** $= \log_{10} (1.5 \times 100)$

$= \log_{10} 1.5 + \log_{10} 100$

$= b - a + 2$

**2** **a** $\log_3 x = \frac{5}{4}$

$x = 3^{\frac{5}{4}} = 3.95$ (3sf)

**b** $3 \log_3 x - 5 \log_3 x = 4$

$\log_3 x = -2$

$x = 3^{-2} = \frac{1}{9}$

**3** **a** **i** $= \log_2 q^{\frac{1}{2}} = \frac{1}{2}\log_2 q = \frac{1}{2}p$

**ii** $= \log_2 8 + \log_2 q = 3 + p$

**b** $3 + p - \frac{1}{2}p = 2$

$p = \log_2 q = -2$

$\therefore\ q = 2^{-2} = \frac{1}{4}$

**4** $2000 = 1000 \times 1.022^{4t}$

$2 = 1.022^{4t}$

$4t \lg 1.022 = \lg 2$

$t = \frac{\lg 2}{4 \lg 1.022} = 7.96$

$\therefore$ 8 years

**5** **a** $(0, -3)$

**b** $k = -4$

**c** $(\frac{1}{3})^x - 4 = 0$

$(\frac{1}{3})^x = 4$

$x = \frac{\lg 4}{\lg \frac{1}{3}} = -1.26$ (3sf)

**6** **a** $\log_3 \frac{x+1}{x-2} = 1$

$\frac{x+1}{x-2} = 3$

$x + 1 = 3x - 6$

$x = \frac{7}{2}$

**b** $(2x + 1) \lg 3 = (x - 4) \lg 2$

$x (\lg 2 - 2 \lg 3) = \lg 3 + 4 \lg 2$

$x = \frac{\lg 3 + 4 \lg 2}{\lg 2 - 2 \lg 3}$

**7** **a** **i** $= 2^{-1}(2^x) = \frac{1}{2}t$

**ii** $= 2(2^{2x}) = 2(2^x)^2 = 2t^2$

**b** $2t^2 - 7t + 6 = 0$

$(2t - 3)(t - 2) = 0$

$t = 2^x = \frac{3}{2}, 2$

$x = \frac{\lg \frac{3}{2}}{\lg 2}, 1 = 0.585$ (3sf), 1

**8** **a** $\log_2 (3x + 5) + 3 = 7$

$3x + 5 = 2^4 = 16$

$x = \frac{11}{3}$

**b** $\log_2 (x + 1) + \log_2 (3x - 1) = 5$

$(x + 1)(3x - 1) = 2^5 = 32$

$3x^2 + 2x - 33 = 0$

$(3x + 11)(x - 3) = 0$

$x = -\frac{11}{3}, 3$

for real $\log_2 (3x - 1)$, $x > \frac{1}{3}$ $\therefore\ x = 3$

**9** **a** $x + 4 = \frac{5}{4}x$

$x = 16$

**b** $y + 2 = \frac{12}{y+1}$

$(y + 2)(y + 1) = 12$

$y^2 + 3y - 10 = 0$

$(y + 5)(y - 2) = 0$

$y > 0 \;\therefore\; y = 2$

**c** $\log_y x = \log_2 16 = 4$

**10** **a** $t = 0 \;\Rightarrow\; n = 2000$

**b** $3600 = \frac{18000}{1+8c^{-3}}$

$1 + 8c^{-3} = 5$

$c^{-3} = \frac{1}{2}$

$c^3 = 2$

$c = \sqrt[3]{2}$

**c** $4000 = \frac{18000}{1+8c^{-t}}$

$1 + 8c^{-t} = \frac{9}{2}$

$c^{-t} = \frac{7}{16}$

$-t = \frac{\lg \frac{7}{16}}{\lg \sqrt[3]{2}}$

$t = 3.578$ weeks = 25 days

**11** **a** **i** $\log_8 x^2 = 2\log_8 x = 2y$

**ii** $y = \log_8 x \;\Rightarrow\; x = 8^y = 2^{3y}$

$\therefore\; \log_2 x = 3y$

**b** $3(2y) + 3y = 6$

$y = \log_8 x = \frac{2}{3}$

$\therefore\; x = 8^{\frac{2}{3}} = 4$

**12** $\log_2 y - \log_2 (3 - 2x) = 1 \;\Rightarrow\; \frac{y}{3-2x} = 2$

$\Rightarrow\; y = 6 - 4x$

$\log_4 xy = \frac{1}{2} \;\Rightarrow\; xy = 4^{\frac{1}{2}} = 2$

sub. $x(6 - 4x) = 2$

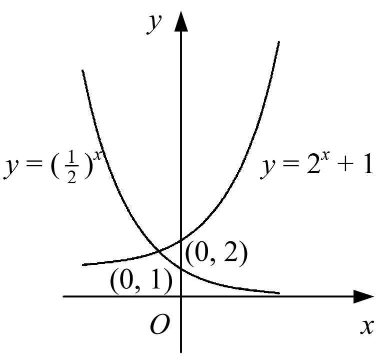
$2x^2 - 3x + 1 = 0$

$(2x - 1)(x - 1) = 0$

$x = \frac{1}{2}, 1$

$\therefore\; x = \frac{1}{2}, y = 4$ or $x = 1, y = 2$

**13** **a**




**b** at $A$, $2^x + 1 = (\frac{1}{2})^x$

$(2^x)^2 + 2^x = 1$

$2^{2x} + 2^x - 1 = 0$

**c** $2^x = \frac{-1 \pm \sqrt{1+4}}{2}$

$2^x = \frac{-1-\sqrt{5}}{2}$ [no sols] or $\frac{-1+\sqrt{5}}{2}$

$\therefore\; 2^x = \frac{1}{2}\sqrt{5} - \frac{1}{2}$

$\therefore\; y = (\frac{1}{2}\sqrt{5} - \frac{1}{2}) + 1 = \frac{1}{2}(\sqrt{5} + 1)$

**14** **a** when $x = 1$,

LHS = $8 - 4(4) + 2 + 6 = 0$

$\therefore\; x = 1$ is a solution

**b** $2^{3x} = (2^x)^3 = u^3$

$2^{2x} = (2^x)^2 = u^2$

$\therefore$ (I) $\Rightarrow\; u^3 - 4u^2 + u + 6 = 0$

**c** $x = 1 \;\Rightarrow\; u = 2 \;\therefore\; (u - 2)$ is a factor

$$
\begin{array}{r|l}
 & u^2 - 2u - 3 \\
u - 2 & u^3 - 4u^2 + u + 6 \\
 & u^3 - 2u^2 \\
 & \quad - 2u^2 + u \\
 & \quad - 2u^2 + 4u \\
 & \qquad - 3u + 6 \\
 & \qquad - 3u + 6
\end{array}
$$

$(u - 2)(u^2 - 2u - 3) = 0$

$(u - 2)(u - 3)(u + 1) = 0$

$u = 2^x = -1$ [no sols], 2 or 3

$x = 1$ (given) or $\frac{\lg 3}{\lg 2} = 1.58$