Math Studies Practice Answers.

36) $f(x) = 2x^3 - 5x^2 + 3x + 1$

a) $f'(x) = 6x^2 - 10x + 3$ //

b) $f'(2) = 6(2)^2 - 10(2) + 3$

$= 6(4) - 20 + 3$

$= 24 - 20 + 3$

$= 7$ //

c) Equation of the tangent to the curve of $y = f(x)$

@ $(2, 3)$ $x = 2$, $y = 3$

$m = f'(2) = 7$

$y = mx + b$

$3 = (7)(2) + b$

$3 = 14 + b$

$b = -11$

$\therefore$ Equation of the tangent

$y = 7x - 11$ //

37) $f(x) = \frac{1}{2}x^3 - 2x^2 + 3$

a) $f'(x) = \frac{3}{2}x^2 - 4x$

b) Omit.

c) Find equation of tangent to the curve @ $(1, 1.5)$

$x = 1$, $y = 1.5$ or $\frac{3}{2}$

$m = f'(1) = \frac{3}{2}(1) - 4(1)$

$= \frac{3}{2} - \frac{8}{2}$

$= -\frac{5}{2}$

$y = mx + b$

$\frac{3}{2} = (-\frac{5}{2})(1) + b$

$\frac{3}{2} + \frac{5}{2} = b$

$\frac{8}{2} = b$

$b = 4$

$\therefore$ Equation of the tangent

$y = -\frac{5}{2}x + 4$ //

38)

x			$1<x<4$	4	$4\le x\le 7$
$f'(x)$			negative (decreasing)	0	increasing

$1<x<4$ $\quad x=4 \quad$ $4\le x\le 7$

@ point $P(4,2)$ $\quad f'(x)=0$

a) $y=mx+b \qquad y=2,\ x=4$

$(2)=(0)(4)+b \qquad m=0$

$b=2$

$\therefore$ The equation tangent to $f(x)$ at P is

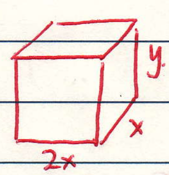
$y=2$ //

(horizontal line).

b) Since at $1<x<4$ $f'(x)$ is negative, meaning the value is decreasing, for $f(x)$. At $f(4)$, the graph will change direction, meaning it will be a local minimum. Therefore $f(4)$ should be less than $f(2)$

c) Point P is the local minimum

39)



$SA = 300\text{ cm}^2$.

$SA = 2(lw) + 2(wh) + 2(lh)$

a) $SA = 2(2x)(x) + 2(xy) + 2(2x)(y)$

$= 4x^2 + 4xy + 2xy$

$SA = 4x^2 + 6xy$ //

$\therefore 4x^2 + 6xy = 300$

b) write y in terms of x

$\dfrac{6xy}{6x} = \dfrac{300}{6x} - \dfrac{4x^2}{6x}$

$y = \dfrac{50}{x} - \dfrac{2x}{3}$

$V = lwh$

c) Volume $= (2x)(x)(y)$

$V = (2x)(x)\left(\dfrac{50}{x} - \dfrac{2x}{3}\right)$

$= 2x^2\left(\dfrac{50}{x} - \dfrac{2x}{3}\right)$

$= \dfrac{100x^2}{x} - \dfrac{4x^3}{3}$

$V = 100x - \dfrac{4}{3}x^3$ //

d) $\dfrac{dV}{dx} = 100 - 4x^2$ //

e) i) max = when $\dfrac{dV}{dx} = 0$

$0 = 100 - 4x^2$

$-100 = -4x^2$

$25 = x^2$

$x = \pm 5$, $x=5$

$y = \dfrac{50}{5} - \dfrac{2(5)}{3}$

$y = 10 - \dfrac{10}{3} = \dfrac{20}{3}$ //

ii) Max volume

$V = 100(5) - \frac{4}{3}(5)^3$

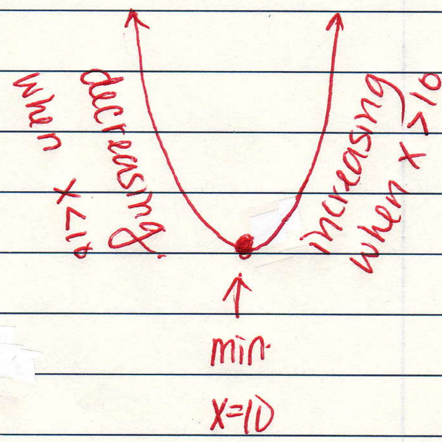
$= 500 - \frac{500}{3}$

$= \frac{1000}{3}$

$= 333.33\text{ cm}^2$ //

4a) $C(x) = x + \frac{100}{x}$.

a) find $C'(x)$



$C(x) = x + 100x^{-1}$

$C'(x) = 1 - 100x^{-2}$

$C'(x) = 1 - \frac{100}{x^2}$

b) if $C(10)$ is a minimum:

→ $C'(10) = 0$

→ $C'(x) < 0$ when $x < 10$

→ $C'(x) > 0$ when $x > 10$

x	$x<10$	$x=10$	$x>10$
$C'(x)$	$C'(1) = 1 - \frac{100}{1}$ $= -99$.	$C'(10) = 1 - \frac{100}{(10)^2}$ $= 1-1=0$	$C'(11) = 1 - \frac{100}{121}$ $= \frac{21}{121}$
behaviour	decreasing.	stationary point. (minimum)	increasing.

∴ Since $C'(x)$ when $x<10$ is positive and $C'(x)$ when $x>10$ is negative, thus $C(10)$ is the minimum

∴ The cost per person is a minimum when 10 people are invited to the party.

c) Calculate min cost ∴ find $C(10)$

$C(10) = 10 + \frac{100}{10}$

$= 10 + 10$

$= 20$

∴ The minimum cost is \$20.