Mixed Numbers

To convert a mixed number, $5\frac{2}{7}$, to an improper fraction, $\frac{37}{7}$:

Numerator

Whole Number $\longrightarrow$ $5\frac{2}{7} = \frac{37}{7}$
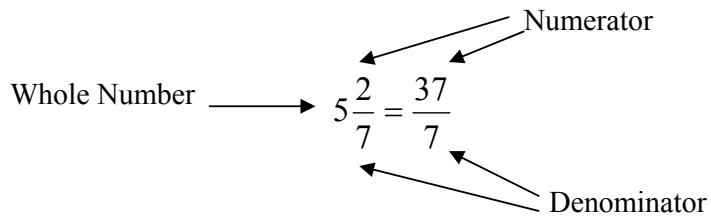
Denominator

$5\frac{2}{7}$	Work in a clockwise direction, beginning with the denominator, (7).
5 x 7 = 35	Multiply the denominator (7) by the whole number, (5)
35 + 2 = 37	Add that product, (35), to the numerator (2) of the fraction.
$\frac{(5\times 7)+2}{7} = \frac{37}{7}$	The denominator remains the same for the mixed number and the improper fraction.

Convert to Improper Fractions:

1) $4\frac{2}{5} =$

2) $5\frac{3}{8} =$

3) $2\frac{4}{9} =$

4) $5\frac{6}{7} =$

5) $8\frac{1}{8} =$

6) $14\frac{3}{4} =$

7) $6\frac{3}{5} =$

8) $9\frac{1}{10} =$

9) $16\frac{1}{2} =$

10) $8\frac{0}{1} =$

11) $9 =$
Hint: See #10

12) $7\frac{3}{4} =$

13) $12\frac{5}{9} =$

14) $10\frac{3}{8} =$

15) $28\frac{2}{3} =$

Finding Equivalent Fractions with Larger Denominators

This process is sometimes called "**Boosting**"

$$Example: \frac{5}{8} = \frac{?}{56}$$

$56 \div 8 = 7$ Divide the larger denominator by the smaller to find the factor used to multiply the denominator. (Note: The product of the smaller denominator and the factor is the larger denominator)

$\frac{5}{8} \times \frac{7}{7} = \frac{5 \times 7}{8 \times 7}$ Use this factor to multiply the numerator.

$\frac{5}{8} = \frac{35}{56}$ The result is two equivalent fractions.

Note: Equal denominators are required for addition and subtraction of fractions.

Find the equivalent fractions as indicated:

1) $\frac{2}{5} = \frac{}{15}$

2) $\frac{3}{8} = \frac{}{32}$

3) $\frac{4}{9} = \frac{}{54}$

4) $\frac{6}{7} = \frac{}{49}$

5) $\frac{1}{8} = \frac{}{48}$

6) $\frac{3}{4} = \frac{}{44}$

7) $\frac{3}{5} = \frac{}{45}$

8) $\frac{1}{10} = \frac{}{60}$

9) $\frac{1}{2} = \frac{}{28}$

10) $\frac{10}{100} = \frac{}{700}$

11) $\frac{8}{9} = \frac{}{81}$

12) $\frac{3}{4} = \frac{}{68}$

13) $\frac{5}{9} = \frac{}{108}$

14) $\frac{3}{8} = \frac{}{112}$

15) $\frac{2}{3} = \frac{}{462}$

Equivalent Fractions with Smaller Denominators
Reducing Fractions

Example: Reduce the following fraction to lowest terms

$$\frac{90}{105}$$

There are **three** common **methods,** DO NOT mix steps of the methods!

Method 1:	
$\dfrac{90 \div 15}{105 \div 15} = \dfrac{6}{7}$	The Greatest Common Factor for 90 and 105 is 15. Divide the numerator and the denominator by the GCF, 15.

Method 2:	
	Examine the numerator and denominator for any common factors, divide both numerator and denominator by that common factor. Repeat as needed.
$\dfrac{90 \div 5}{105 \div 5} = \dfrac{18}{21}$	➢ Both 90 and 105 are divisible by 5.
$\dfrac{18 \div 3}{21 \div 3} = \dfrac{6}{7}$	➢ Both 18 and 21 are divisible by 3.

Method 3:	
$\dfrac{90}{105} = \dfrac{2 \times 3 \times 3 \times 5}{7 \times 3 \times 5}$	Express the numerator and denominator as a product of prime factors.
$\dfrac{90}{105} = \dfrac{2 \times 3 \times (3 \times 5)}{7 \times (3 \times 5)}$	Divide numerator and denominator by common factors, (3x5)
$= \dfrac{2 \times 3}{7} = \dfrac{6}{7}$	Multiply remaining factors.

Reduce these fractions.

1) $\dfrac{28}{50} =$

2) $\dfrac{8}{24} =$

3) $\dfrac{30}{54} =$

4) $\dfrac{18}{42} =$

5) $\dfrac{32}{48} =$

6) $\dfrac{36}{54} =$

7) $\dfrac{14}{56} =$

8) $\dfrac{18}{28} =$

9) $\dfrac{36}{216} =$

10) $\dfrac{35}{42} =$

11) $12\dfrac{54}{99} =$

12) $15\dfrac{280}{320} =$

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Improper Fractions

Example: Convert $\dfrac{14}{3}$ to an Improper Fraction

$14 \div 3 = 4$ Remember: Dividend $\div$ Divisor = Quotient

Remainder 2   Divide the numerator (14) by the denominator (3).

$\dfrac{14}{3} = 4\dfrac{2}{3}$ Write the mixed number in the form: $Quotient\dfrac{remainder}{divisor}$

Note: Check you answer to see if you can reduce the fraction.

Convert these improper fractions to mixed numbers. *Be sure to reduce when it's possible.*

#11, 12 Hint: how many wholes will there be?

1) $\dfrac{8}{5}$ =

2) $\dfrac{18}{7}$ =

3) $\dfrac{37}{9}$ =

4) $\dfrac{127}{5}$ =

5) $\dfrac{32}{9}$ =

6) $\dfrac{114}{5}$ =

7) $\dfrac{128}{3}$ =

8) $\dfrac{401}{3}$ =

9) $\dfrac{36}{6}$ =

10) $\dfrac{235}{2}$ =

11) $15\dfrac{280}{6}$ =

12) $8\dfrac{315}{3}$ =

13) $\dfrac{54}{8}$ =

14) $\dfrac{26}{8}$ =

15) $\dfrac{258}{9}$ =

Least Common Multiple (LCM)
Used to find the Least Common Denominator (LCD)

Example: Find the LCM of 30 and 45

Note: There are **four** common **methods**; DO NOT mix the steps of the methods!

Method 1	
	Remember that multiples are equal to or larger than the given number.
30, 60, **90**, 120, …	List the multiples of each of the given numbers, in
45, **90**, 135, …	ascending order.
LCM = 90	The LCM is the first multiple common to both lists.

Method 2	
45, 90, 135, …	List the multiples of the larger number.
$45 \div 30$ remainder	Divide each in turn by the smaller.
$90 \div 30$ no remainder	The LCM is the multiple that the smaller number
LCM = 90	divides without leaving a remainder.

Method 3	
$30 \div 5 = 6$; $45 \div 5 = 9$	Divide both numbers by any common factor, (5 then
$6 \div 3 = 2$; $9 \div 3 = 3$	3). Continue until there are no more common factors.
	Note: 2 and 3, the results of the last division have no common factors.
LCM = $5 \times 3 \times 2 \times 3$	The LCM equals the product of the factors, (5 and
= 90	3) and the remaining quotients, (2 and 3).

Method 4	
30　　　　　45	
5 x 6　　　5 x 9	Find the prime factors of each the given numbers.
5 x 2 x 3　　5 x 3 x 3	
$30 = 5 \times 2 \times 3$	Write each number as a product of primes using
$45 = 5 \times 3 \times 3$ **Or** $45 = 5 \times 3^2$	exponents, if required.
LCM = $2 \times 3^2 \times 5$	LCM equals the product of all the factors to the
= 90	highest power.

In each exercise, find the LCM of the given numbers.

1)   4   and   18   7)   50   and   75

2)   16   and   40   8)   24   and   30

3)   20   and   28   9)   36   and   45

4)   5   and   8   10)   8   and   20

5)   12   and   18   11)   16   and   20

                                      12)   28,   35,   and   21

6)   12   and   16

Addition and Subtraction of Fractions
with the Same Denominator

To add or subtract fractions, the denominators MUST be the same.

Example 1:

$$\frac{3}{5} - \frac{1}{5} = ?$$

$$\frac{3}{5} - \frac{1}{5} = \frac{3-1}{5}$$

Because both fractions have the same denominator, you may subtract the numerators and keep the denominator.

$$= \frac{2}{5}$$

Example 2:

$$\frac{5}{9} + \frac{7}{9} = ?$$

$$\frac{5}{9} + \frac{7}{9} = \frac{5+7}{9}$$

Because both fractions have the same denominator, you may add the numerators and keep the denominator.

$$= \frac{12}{9}$$

$$= 1\frac{3}{9}$$

Always change improper fractions to a mixed number.

$$= 1\frac{1}{3}$$

Reduce, when possible.

Add or Subtract as indicated.

1. $\dfrac{4}{8} + \dfrac{3}{8}$

2. $\dfrac{7}{10} - \dfrac{1}{10}$

3. $\dfrac{7}{48} + \dfrac{9}{48} + \dfrac{4}{48}$

4. $\dfrac{40}{37} - \dfrac{3}{37}$

5. $\dfrac{10}{13} + \dfrac{4}{13}$

6. $\dfrac{9}{17} + \dfrac{11}{17} + \dfrac{17}{17}$

7. $\dfrac{2}{3} + \dfrac{4}{3} - \dfrac{6}{3}$

8. $\dfrac{7}{6} - \dfrac{5}{6} + \dfrac{1}{6}$

9. $\dfrac{7}{13} + \dfrac{9}{13}$

Addition and Subtraction of Fractions
with Different Denominators

Remember: In order to add or subtract fractions, the denominators MUST be the same.

Example:

$$\frac{2}{3} + \frac{3}{8} = ?$$

LCM = 24   Find the LCM

$$\frac{2}{3} \times \frac{8}{8} = \frac{16}{24}$$ Write the problem vertically.

$$+ \ \frac{3}{8} \times \frac{3}{3} = \frac{9}{24}$$ Find the equivalent fractions with the LCM as a denominator.

$$\frac{25}{24}$$ Add the fractions with the same denominator.

$$\frac{25}{24} = 1\frac{1}{24}$$ Remember to write as a mixed number and reduce when possible!

Add or Subtract:

1) $\frac{7}{8} + \frac{3}{4}$

2) $\frac{7}{8} - \frac{3}{4}$

3) $\frac{11}{12} + \frac{17}{18}$

4) $\frac{3}{7} + \frac{2}{5}$

5) $\frac{15}{24} - \frac{10}{27}$

6) $\frac{7}{12} + \frac{5}{16}$

7) $\frac{16}{27} - \frac{5}{24}$

8) $1\frac{1}{4} + \frac{3}{8}$

9) $\frac{11}{4} + \frac{23}{18}$

10) $\frac{29}{8} + \frac{9}{7}$

11) $2\frac{13}{35} - 1\frac{5}{14}$

12) $\frac{2}{3} + \frac{1}{21} - \frac{2}{7}$

Subtraction of Fractions with Borrowing

Example 1:

$$7 - 1\frac{1}{3} = ?$$

Example 2:

$$5\frac{1}{3} - 2\frac{5}{6} = ?$$

Note: There are two common methods; DO NOT mix the steps of the methods!

Method 1 *Example 1* $7 = 6\frac{3}{3}$ $-\ 1\frac{1}{3} = 1\frac{1}{3}$ $\qquad\quad 5\frac{2}{3}$	**Subtraction with Borrowing** Write problem vertically Cannot subtract fraction from whole without finding common denominator. Borrow one whole from 7 and express as $\dfrac{LCD}{LCD}$. $\left(1 = \dfrac{3}{3}\right)$ Subtract numerators and whole numbers.
Example 2 $5\frac{1}{3} = 5\frac{2}{6} = 4\frac{8}{6}$ $-\ 2\frac{5}{6} = 2\frac{5}{6} = 2\frac{5}{6}$ $\qquad\qquad\quad 2\frac{3}{6} = 2\frac{1}{2}$	Write problem vertically and find LCD Cannot subtract 5 from 2. Borrow one whole from 5, $\left(4\dfrac{6}{6}\right)$ and add $\left(5\dfrac{2}{6} = 4\dfrac{6+2}{6}\right)$. Subtract numerators and whole numbers; reduce as needed.
Method 2 *Example 1:* $7 = \dfrac{21}{3}$ $-\ 1\frac{1}{3} = \dfrac{4}{3}$ $\qquad\quad \dfrac{17}{3} = 5\frac{2}{3}$	**Subtraction Using Improper Fractions** Write the problem vertically. Convert the whole numbers and mixed numbers to improper fractions using the LCD. Subtract $\left(\dfrac{21-4}{3}\right)$ and convert improper fraction to mixed number.
Example 2: $5\frac{1}{3} = 5\frac{2}{6} = \dfrac{32}{6}$ $-\ 2\frac{5}{6} = 2\frac{5}{6} = \dfrac{17}{6}$ $\qquad\qquad\quad \dfrac{15}{6} = 2\frac{3}{2}$ $\qquad\qquad\quad 2\frac{3}{2} = 2\frac{1}{2}$	Write problem vertically and find the LCD. Change the mixed numbers to improper fractions. Subtract the numerators. Convert to a mixed number. Reduce.

Subtract:

1) $5 - 2\frac{1}{3}$

2) $7 - 1\frac{1}{6}$

3) $10 - 4\frac{5}{6}$

4) $3\frac{5}{8} - 2\frac{7}{8}$

5) $1\frac{1}{8} - \frac{3}{4}$

6) $3\frac{5}{12} - 1\frac{15}{16}$

7) $8 - 6\frac{4}{5}$

8) $4\frac{3}{8} - 3\frac{5}{6}$

9) $17 - 4\frac{5}{9}$

10) $5\frac{5}{18} - 1\frac{3}{4}$

11) $5\frac{2}{7} - 3\frac{3}{8}$

12) $18 - 1\frac{7}{16} - \frac{7}{12}$

Multiplication of Fractions

Example:

$$\frac{3}{10} \times 3\frac{5}{6}$$

Note: LCD is not needed to multiply fractions.

$3\frac{5}{6} = \frac{(6 \times 3) + 5}{6}$	Change mixed numbers to improper fractions
$\frac{3}{10} \times \frac{23}{6} = \frac{1 \times 23}{10 \times 2}$	Before multiplying, reduce by dividing any numerator with any denominator with a common factor. (3 and 6 have a common factor of 3)
$\frac{1 \times 23}{10 \times 2} = \frac{23}{20}$	Multiply numerators and denominators
$\frac{23}{20} = 1\frac{3}{20}$	Convert improper fractions to mixed numbers.

Multiply:

1) $4\frac{1}{2} \times \frac{2}{3}$

2) $3\frac{1}{5} \times 1\frac{1}{4}$

3) $6 \times 1\frac{1}{9}$

4) $2\frac{1}{6} \times 1\frac{1}{2}$

5) $\frac{10}{11} \times 1\frac{7}{15}$

6) $4\frac{3}{5} \times 15$

7) $3\frac{3}{8} \times 2\frac{2}{9}$

8) $34 \times 2\frac{3}{17}$

9) $9\frac{7}{8} \times \frac{4}{5}$

10) $7\frac{9}{10} \times 1\frac{1}{4}$

11) $18 \times 1\frac{3}{7} \times \frac{4}{15}$

12) $3\frac{1}{5} \times 1\frac{5}{6} \times \frac{3}{8}$

Division of Fractions

Example:

$$2\frac{3}{4} \div 2\frac{3}{8} \qquad \textbf{OR} \qquad \frac{2\frac{3}{4}}{2\frac{3}{8}}$$

Note: One fraction divided by another may be expressed in either way shown above. Also, LCD is not needed to divide fractions.

$2\frac{3}{4} = \frac{11}{4}$ and $2\frac{3}{8} = \frac{19}{8}$ — Convert mixed numbers to improper fractions

$\dfrac{11}{4} \div \dfrac{19}{8} = \dfrac{11}{4} \times \dfrac{8}{19}$ — Invert the divisor $\left(\dfrac{19}{8}\right)$. (Turn the fraction after the division sign upside down)

$\dfrac{11 \times 8}{4 \times 19} = \dfrac{11 \times 2}{1 \times 19}$ — Reduce if possible. (4 and 8 have a common factor)

$\dfrac{11 \times 2}{1 \times 19} = \dfrac{22}{19}$ — Multiply numerators and denominators

$\dfrac{22}{19} = 1\dfrac{3}{19}$ — Convert to a mixed number and reduce if needed.

Divide these fractions.   Reduce to lowest terms!

1) $\dfrac{5}{6} \div \dfrac{1}{2}$

4) $\dfrac{\frac{1}{2}}{\frac{1}{3}} =$

7) $3\frac{1}{7} \div 2\frac{5}{14} =$

2) $\dfrac{3}{4} \div \dfrac{3}{7} =$

5) $\dfrac{1}{2} \div 6 =$

8) $\dfrac{2\frac{5}{8}}{1\frac{7}{8}}$

3) $3 \div 1\frac{2}{5} =$

6) $2\frac{1}{4} \div 3 =$

9) $4\frac{1}{2} \div 1\frac{3}{4} =$

Some Fraction Word Problems

Example 1:

One day Ashley biked $\frac{3}{4}$ of a mile before lunch and $\frac{7}{8}$ of a mile after lunch. How far did she cycle that day?

Note: this problem is asking you to add the distances traveled.

$\frac{3}{4} + \frac{7}{8}$ To add fractions, find a LCD (8).

Add the numerators; keep the denominators.

$\frac{6}{8} + \frac{7}{8}$

Convert improper fraction to a mixed number; reduce if needed.

$\frac{13}{8} = 1\frac{5}{8}$

Ashley cycled $1\frac{5}{8}$ miles that day.

Example 2:

A tailor needs $3\frac{1}{4}$ yards of fabric to make a jacket. How many jackets can he make with $19\frac{1}{2}$ yards of fabric?

Note: this problem is asking you to divide.

$19\frac{1}{2} \div 3\frac{1}{4}$

To divide fractions, convert mixed numbers to improper fractions.

$\frac{39}{2} \div \frac{13}{4}$

Invert the divisor and reduce if possible, (39 and 13 have a common factor, as do 2 and 4).

$\frac{39}{2} \times \frac{4}{13} = \frac{3 \times 2}{1 \times 1}$

Multiply numerators and denominators.

$\frac{3}{1} = 3$

The tailor can make 3 jackets from $19\frac{1}{2}$ yards of fabric.

Solve the following problems.

1. An empty box weighs $2\frac{1}{4}$ pounds. It is then filled with $16\frac{2}{3}$ pounds of fruit. What is the weight of the box when it is full?

2. Yanni is making formula for the baby. Each bottle contains $6\frac{2}{5}$ scoops of formula. The formula container holds 320 scoops of formula. How many bottles of formula can Yanni make?

3. Miguel bought $2\frac{1}{4}$ pounds of hamburger, $1\frac{1}{5}$ pounds of sliced turkey, and 2 pounds of cheese. What was the total weight of all of his purchases?

4. Sheila had 8 yards of fabric. She used $2\frac{1}{4}$ yards to make a dress. How much fabric does she have left?

5. A father leaves his money to his four children. The first received $\frac{1}{3}$, the second received $\frac{1}{6}$, and the third received $\frac{2}{5}$. How much did the remaining child receive? (Hint: You can think of father's money as one whole.)

6. Find the total perimeter (sum of the sides) of an equilateral triangle, (triangle with equal sides), if each side measures $2\frac{1}{4}$ inches.

Answers to Fractions Competency Packet

p. 6		p. 7		p. 8		p. 9		p. 11	
1)	$\frac{22}{5}$	1)	6	1)	$\frac{14}{25}$	1)	$1\frac{3}{5}$	1)	36
2)	$\frac{43}{8}$	2)	12	2)	$\frac{1}{3}$	2)	$2\frac{4}{7}$	2)	80
3)	$\frac{22}{9}$	3)	24	3)	$\frac{5}{9}$	3)	$4\frac{1}{9}$	3)	140
4)	$\frac{41}{7}$	4)	42	4)	$\frac{3}{7}$	4)	$25\frac{2}{5}$	4)	40
5)	$\frac{65}{8}$	5)	6	5)	$\frac{2}{3}$	5)	$3\frac{5}{9}$	5)	36
6)	$\frac{59}{4}$	6)	33	6)	$\frac{2}{3}$	6)	$22\frac{4}{5}$	6)	48
7)	$\frac{33}{5}$	7)	27	7)	$\frac{1}{4}$	7)	$42\frac{2}{3}$	7)	150
8)	$\frac{91}{10}$	8)	6	8)	$\frac{9}{14}$	8)	$133\frac{2}{3}$	8)	120
9)	$\frac{33}{2}$	9)	14	9)	$\frac{1}{6}$	9)	6	9)	180
10)	$\frac{8}{1}$	10)	70	10)	$\frac{5}{6}$	10)	$117\frac{1}{2}$	10)	40
11)	$\frac{9}{1}$	11)	72	11)	$12\frac{6}{11}$	11)	$61\frac{2}{3}$	11)	80
12)	$\frac{31}{4}$	12)	51	12)	$15\frac{7}{8}$	12)	113	12)	420
13)	$\frac{113}{9}$	13)	60			13)	$6\frac{3}{4}$		
14)	$\frac{83}{8}$	14)	42			14)	$3\frac{1}{4}$		
15)	$\frac{86}{3}$	15)	308			15)	$28\frac{2}{3}$		

p. 12	p. 13	p. 15	p. 16	p. 17
1) $\dfrac{7}{8}$	1) $1\dfrac{5}{8}$	1) $2\dfrac{2}{3}$	1) 3	1) $1\dfrac{2}{3}$
2) $\dfrac{3}{5}$	2) $\dfrac{1}{8}$	2) $5\dfrac{5}{6}$	2) 4	2) $1\dfrac{3}{4}$
3) $\dfrac{5}{12}$	3) $1\dfrac{31}{36}$	3) $5\dfrac{1}{6}$	3) $6\dfrac{2}{3}$	3) $2\dfrac{1}{7}$
4) 1	4) $\dfrac{29}{35}$	4) $\dfrac{3}{4}$	4) $3\dfrac{1}{4}$	4) $1\dfrac{1}{2}$
5) $1\dfrac{1}{13}$	5) $\dfrac{55}{216}$	5) $\dfrac{3}{8}$	5) $1\dfrac{1}{3}$	5) $\dfrac{1}{12}$
6) $2\dfrac{3}{17}$	6) $\dfrac{43}{48}$	6) $1\dfrac{23}{48}$	6) 69	6) $\dfrac{3}{4}$
7) $\dfrac{0}{3}$	7) $\dfrac{83}{216}$	7) $1\dfrac{1}{5}$	7) $7\dfrac{1}{2}$	7) $1\dfrac{1}{3}$
8) $\dfrac{1}{2}$	8) $1\dfrac{5}{8}$	8) $\dfrac{13}{24}$	8) 74	8) $1\dfrac{2}{5}$
9) $1\dfrac{3}{13}$	9) $4\dfrac{1}{36}$	9) $12\dfrac{4}{9}$	9) $7\dfrac{9}{10}$	9) $2\dfrac{4}{7}$
	10) $4\dfrac{51}{56}$	10) $3\dfrac{19}{36}$	10) $9\dfrac{7}{8}$	
	11) $1\dfrac{1}{70}$	11) $1\dfrac{51}{56}$	11) $6\dfrac{6}{7}$	
	12) $\dfrac{3}{7}$	12) $15\dfrac{47}{48}$	12) $2\dfrac{1}{5}$	

P. 19

1) $18\dfrac{11}{12}$ pounds   3) $5\dfrac{9}{20}$ pounds   5) $\dfrac{1}{10}$ of the money

2) 50 bottles   4) $5\dfrac{3}{4}$ yards   6) $6\dfrac{3}{4}$ inches