15 Statistics

Chapter Contents

Learning Outcomes

Students will be able to:

- Work with data arranged in unequal intervals.
- Use standard deviation and the mean to compare sets of data.
- Understand the normal distribution.
- Find a line of best fit for a set of data.
- Use correlation to compare sets of data.

Areas of Interaction

Approaches to Learning (Knowledge Acquisition, Logical Thinking, Communicating, Reflection), Human Ingenuity, Environments

15:01A | Review: Representing Data

In Book 4 you learnt how to represent data in a number of ways:

Frequency Distribution Tables

worked example

The percentage results for sixty students in an examination were:

78	63	89	55	92	74	62	69	43	90	91	83	49	37	58
73	78	65	62	87	95	77	69	82	71	60	61	53	59	42
43	33	98	88	73	82	75	63	67	59	57	48	50	51	66
73	68	46	69	70	91	83	62	47	39	63	67	74	52	78

To organise this data into a table we use **class intervals** or **groups**: 29–37, 38–46 etc.

Class	*Class centre (c.c.)*	*Tally*	*Frequency (f)*	*Cumulative frequency*
29–37	33	||	2	2
38–46	42	~~||||~~	5	7
47–55	51	~~||||~~ |||	8	15
56–64	60	~~||||~~ ~~||||~~ ||	12	27
65–73	69	~~||||~~ ~~||||~~ ||||	14	41
74–82	78	~~||||~~ ||||	9	50
83–91	87	~~||||~~ ||	7	57
92–100	96	|||	3	60
		Totals:	60	

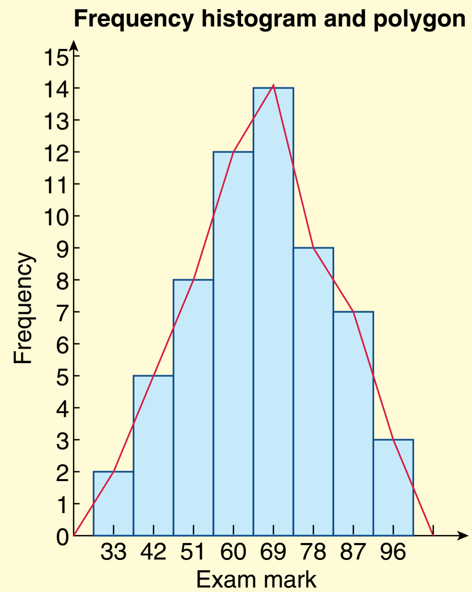
Frequency Histograms and Polygons

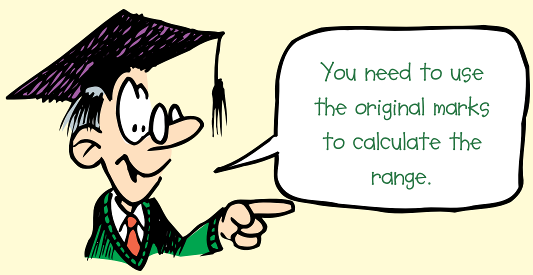
To give a visual representation of the data we can arrange it into a column graph called a **frequency histogram**, and a line graph called a **frequency polygon**.

In these graphs the class centres are used for the middle of the column and to plot the line.

When constructing frequency diagrams for grouped data, the only point to note is that the columns are indicated on the horizontal axis by the class centres. The diagrams for the worked example above would look like these.

- The modal class 65–73 is represented by the class centre 69.
- The **frequency polygon** can be drawn by joining the midpoints of the tops of columns. To complete the polygon, assume that the classes on either side of the columns have zero members.



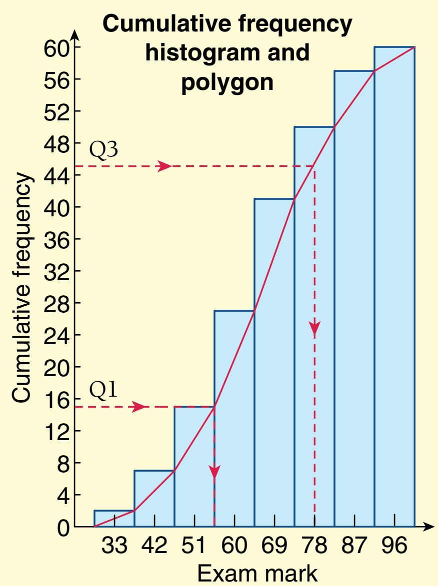


Note: the area under the columns is the same as the area under the line — this is important as it represents the total number of pieces of data — in this case 60.

Cumulative Frequency Histograms and Polygon

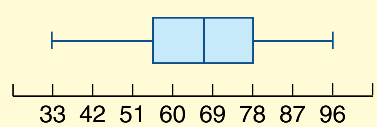
The cumulative frequency histogram is plotted in the same way using the respective numbers from the table. The polygon in this case, however, is a little different.

- The **cumulative frequency polygon** can be drawn by joining the top right corners of each column.
- There are 60 scores altogether so to find the median class we come across from 30 until we meet the polygon and then down to the horizontal axis.
- Clearly the median class is 65–73.
- An estimate of the median mark can be read from the horizontal axis, ie 67.
- The inter-quartile range can be calculated from the horizontal axis by calculating Q3 – Q1 = 78 – 56 = 22.



Box and Whisker Plots

A box and whisker plot gives a visual representation of the spread of the scores by showing the median and the inter-quartile range as shown:



Exercise 15:01A

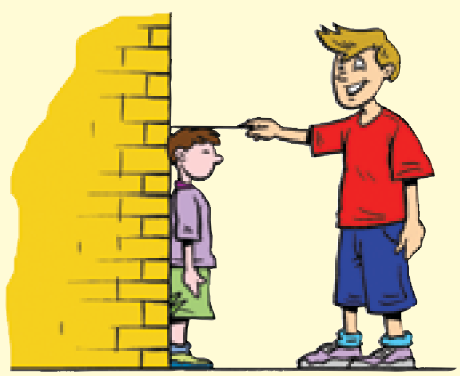
1 The following table shows the distribution of marks in a mathematics test.

Class	*Class centre*	*Frequency*	*Cumulative frequency*
30–34		1	
35–39		3	
40–44		6	
45–49		7	
50–54		6	
55–59		5	
60–64		2	

a Complete the table.
b Construct a frequency histogram and polygon.
c Construct a cumulative frequency histogram and polygon.
d Use your cumulative frequency polygon to estimate:
  i the median **ii** the first quartile Q1
  iii the third quartile Q3 **iv** the inter-quartile range.
e Construct a box and whisker plot of this data.

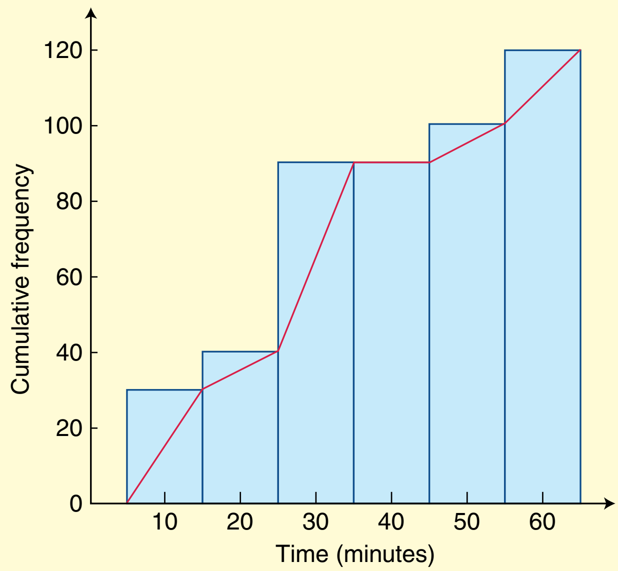
2 The following table shows the heights of 50 Grade 10 boys when measured to the nearest centimetre.

Height (cm)	*Class centre*	*Frequency*	*Cumulative frequency*
146–150			2
151–155			9
156–160			18
161–165			36
166–170			44
171–175			49
176–180			50



a Complete the table.
b Construct a frequency histogram and polygon.
c Construct a cumulative frequency histogram and polygon.
d Use your cumulative frequency polygon to estimate:
  i the median **ii** the first quartile Q1
  iii the third quartile Q3 **iv** the inter-quartile range
e Construct a box and whisker plot of this data.

3 The cumulative frequency histogram below shows the time taken for students in Keishi's grade to travel to school in the morning.



a How many students are in Keishi's grade?

b Complete the following table from the information in the histogram.

Time (min)	*Class centre*	*Frequency*	*Cumulative frequency*
$5 \leqslant x < 15$	10		
$15 \leqslant x < 25$			

c This table has described the groups differently than in the previous two questions. Explain why this might be the case.

d Using the graph, estimate

  i the median

  ii the first quartile Q1

  iii the third quartile Q3

  iv the inter-quartile range

e Construct a box and whisker plot of the data.

4 The table below shows the distance from Bangkok airport to a random sample of international destinations.

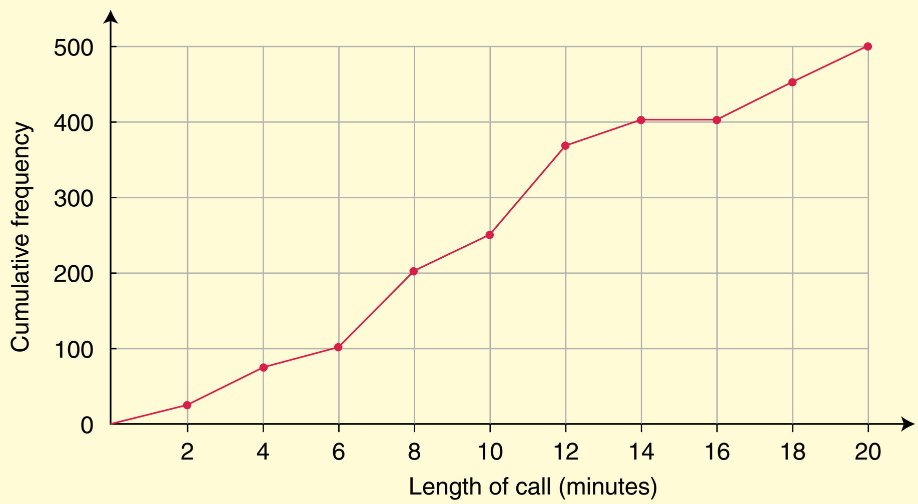
Destination	*Distance (km)*	*Destination*	*Distance (km)*
Auckland	5944	Johannesburg	5574
Athens	4920	Kunming	790
B.S. Begawan	1154	Los Angeles	8260
Brisbane	4522	Madrid	6314
Busan	2305	Muscat	2833
Chengdu	1188	Nagoya	2701
Chittagong	823	New York	8656
Copenhagen	5350	Phnom Penh	329
Dhaka	960	Seoul	2304
Frankfurt	5574	Sydney	4679
Ho Chi Minh	461	Tokyo	2879
Hong Kong	1076	Yangon	363
Isalamabad	2197	Zurich	5608

a Complete this frequency distribution table to summarise the distances.

Distance (km)	*Class centre*	*Frequency*	*Cumulative frequency*
0–			
1000–			
2000–			
3000–			
4000–			
5000–			
6000–			
7000–			
8000–			

b Draw a cumulative frequency polygon to estimate:
  i the median.
  ii the lower quartile Q1
  iii the upper quartile Q3
  iv the inter-quartile range.

5 A telephone exchange records the length (in minutes) of all international phone calls. The cumulative frequency polygon below shows the length of 500 international calls made from Singapore on Christmas Day.



a Arrange this information into a frequency distribution table.
b From the polygon, estimate values for
  i the median.
  ii The lower quartile Q1
  iii The upper quartile Q3
  iv The inter-quartile range.
c Construct a box and whisker plot of the data.

15:01B | Review: Analysing Data

As covered in Book 4, there are a number of measures which help us analyse and interpret data.

Some of these give us information about the location of the **middle** of the data. These are called **measures of central tendency**. These are:

Mean ($\bar{x}$) $= \dfrac{\text{Sum of the scores}}{\text{The number of scores}} = \dfrac{\sum fx}{n}$

Median = middle score when they are arranged in ascending order

Mode = the score (or group of scores) with the highest frequency

Other measures tell us about the spread of the scores. Some of these are better than others:

Range = highest score − lowest score
This only uses 2 scores and does not take into account outlying scores

Inter-quartile range = 3rd quartile − 1st quartile
Although this only measures the spread of the middle 50 per cent of the scores, it still only uses two scores in its calculation.

Standard deviation (σ_n) = average distance of the scores from the mean.
This is the best measure of spread since it uses every score in its calculation.

Measures of central tendency

worked example

Consider the following sets of scores:

Set A: 16, 12, 13, 11, 13, 14, 9, 15, 15, 12

Set B:

Score	38	39	40	41	42	43	44
Frequency	1	4	8	9	4	1	1

To find the mean of the scores we need to find the sum of the scores $(\sum x)$ and the number of scores (n)

For set A

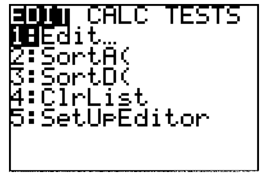
$$\bar{x} = \frac{\sum n}{n} = \frac{16 + 12 + 13 + 11 + 13 + 14 + 9 + 15 + 15 + 12}{10}$$
$$= 13$$

For set B

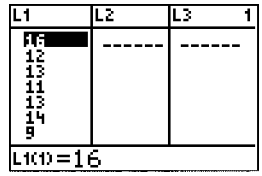
$$\bar{x} = \frac{\sum fx}{n} = \frac{1 \times 38 + 4 \times 39 + 8 \times 40 + 9 \times 41 + 4 \times 42 + 1 \times 43 + 1 \times 44}{28}$$
$$= 40{\cdot}6 \text{ (3 significant figures)}$$

These can also be done on your calculator. The steps required depend on the calculator you use. Here are the steps for the TI-83 or the TI-84.

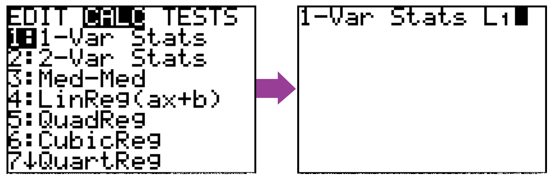
1 Choose statistics by pressing STAT and choosing 1:Edit



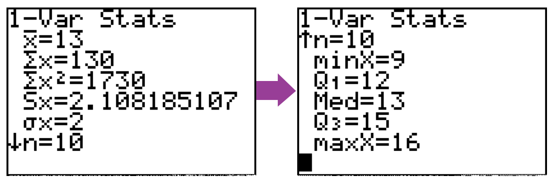
2 This will take you to a table where you need to enter the individual scores. These are for set A.



3 Now go back to STAT and highlight CALC and choose 1: 1-var stat as there is only 1 column of information. You then need to input the column where the information is located. In this case it is in column L1 – or list 1.



4 By scrolling through the next window we can read off the following data:



the mean $\bar{x} = 13$

the sum of the scores $\sum x = 130$

the sum of the scores squared $\sum x^2 = 1730$

the standard deviation of the sample $Sx = 2{\cdot}11$

the standard deviation of the population $\sigma x = 2$

the number of scores $n = 10$

the minimum score $min\ X = 9$

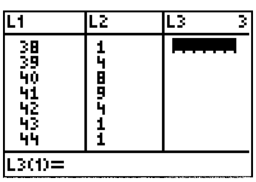
the first quartile $Q_1 = 12$

the median $Med = 13$

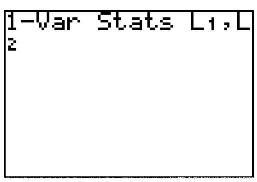
the third quartile $Q_3 = 15$

the maximum score $max\ X = 16$

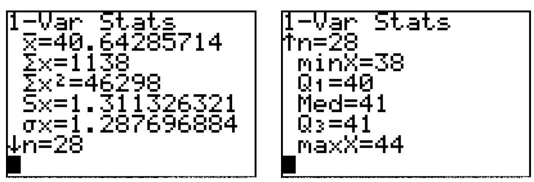
We can do the same for set B by using 2 columns.



Input scores and frequency in two columns.



This time you need to input two columns, the first the score and the second the frequency.



The output is in the same format as before.

continued →→→

This also works for grouped scores; however, the centres must be put in the score column.

To find the mode: This is the score with the highest frequency.

In the case of set A, the mode = 12, 13 and 15 as they all have a frequency of 2.

In the case of set B, the mode = 41 as it has a frequency of 9.

To find the median: This is the middle score when the scores are arranged in ascending order.

In the case of set A: 16, 12, 13, 11, 13, 14, 9, 15, 15, 12

Becomes 9, 11, 12, 12, 13, 13, 14, 15, 15, 16

As there are 10 scores, there are two middle scores: the 5th and 6th. The median is the mean of these two scores: 13.

In the case of set B the scores are already arranged in order.

By adding the cumulative frequency column we get

Score	38	39	40	41	42	43	44
Frequency	1	4	8	9	4	1	1
C. frequency	1	5	13	22	26	27	28

The 14th to the 22nd scores are here.

As there are 28 scores, the 14th and 15th are the middle scores. Both of these are 41. So the median = 41

The median can also be obtained from the calculator as shown above.

For grouped scores, the median must be found from the cumulative frequency polygon as shown in the previous section.

Measures of spread

Suppose we take the same two sets of data:

Set A: 16, 12, 13, 11, 13, 14, 9, 15, 15, 12

Set B:

Score	38	39	40	41	42	43	44
Frequency	1	4	8	9	4	1	1

To find the range: Subtract the lowest score from the highest score

For set A the range = 16 – 9
= 7

For set B the range = 44 – 38
= 14

To find the inter-quartile range: Subtract the lower quartile (Q_1) from the upper quartile (Q_3).

For set A: we must find the scores which mark the quarters

9, 11, 12, 12, 13, 13, 14, 15, 15, 16

Q_1 Q_2 or median Q_3

Inter-quartile range = 15 – 12
= 3

For set B: we again use the cumulative frequency table

Score	38	39	40	41	42	43	44
Frequency	1	4	8	9	4	1	1
C. frequency	1	5	13	22	26	27	28

There are 28 scores, which can be divided into 4 equal lots of 7 scores.

$\therefore Q_1$ = the mean 7th and 8th scores
= 40

$\therefore Q_3$ = the mean 21st and 22nd scores
= 41

Inter-quartile range = 41 – 40
= 1

> ***Note*: the median is another name for the second quartile Q_2**
> **Q_2 = the mean 14th and 15th scores**
> **= 41**

These can also be obtained from your calculator in the same way as the mean.

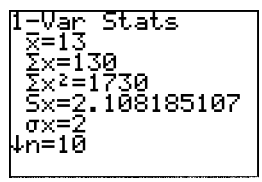
For grouped scores, the quartiles must be found from the cumulative frequency polygon as shown in the previous section.

To find the standard deviation:

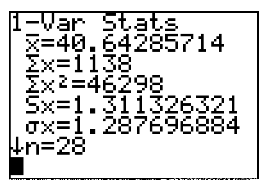
This must be obtained from your calculator by first entering the scores as shown above.

As mentioned before, there are two standard deviations. Since our data is not a sample, but represents the whole population, we use σx for standard deviation.

For set A the standard deviation $\sigma x = 2$



For set B the standard deviation $\sigma x = 1{\cdot}28$



Exercise 15:01B

For each of the questions in exercise 15:01A find

a the mean **b** the mode **c** the range **d** the standard deviation.

investigation 15:01

Investigation 15:01 | Comparing sets of data

Please use the Assessment Grid on page 451 to help you understand what is required for this Investigation.

The object of this investigation is to use different measures to compare sets of scores with each other.

1 Gerui and Maher have a holiday job picking apples. The lists below show how many buckets of apples picked over a 17-day period.

Gerui: 65, 73, 86, 90, 99, 106, 45, 92, 94, 102, 97, 107, 107, 99, 83, 101, 91
Maher: 49, 84, 95, 99, 103, 102, 95, 103, 100, 99, 108, 0, 96, 105, 102, 97, 95

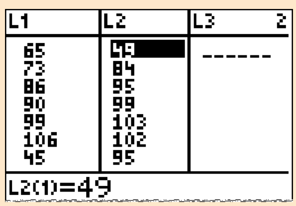
Either by entering these sets of data into your calculator, or by constructing a frequency table, complete the following table.

	Gerui	*Maher*
Mean		
Median		
Range		
Q1		
Q2		
IQR		

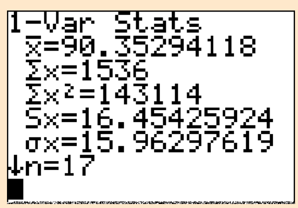
What does this information tell you about the data?
Make a comparison about
i the middles of the sets of data
ii the spread of the data in each set.

TI Calculator Instructions

Using STAT EDIT enter the data.



Calculate the statistics on each set of data.

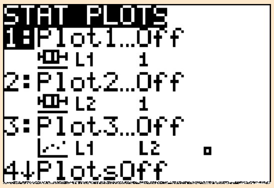


Construct box and whisker plots of both sets of data. This can be done on the TI calculator following the steps below:

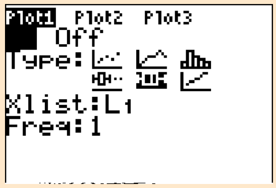
TI Calculator Instructions

You have the data entered into two lists.

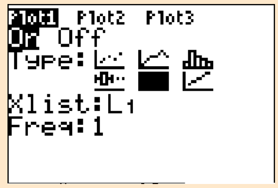
Use STAT PLOT



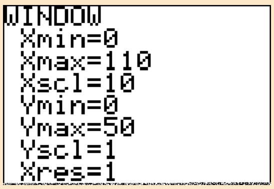
Turn the plot ON



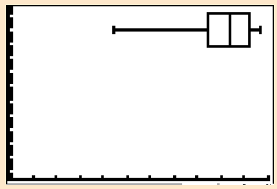
Choose Box and Whisker



Choose an appropriate window



Graph

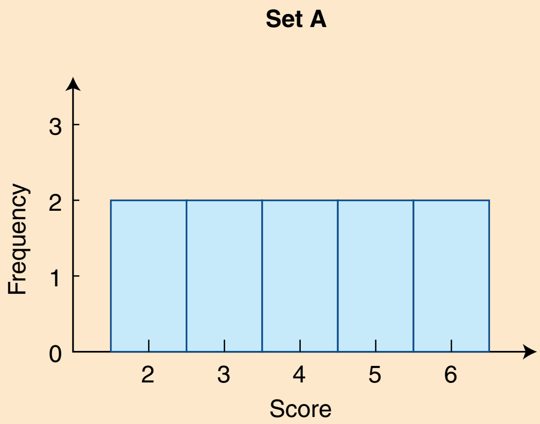


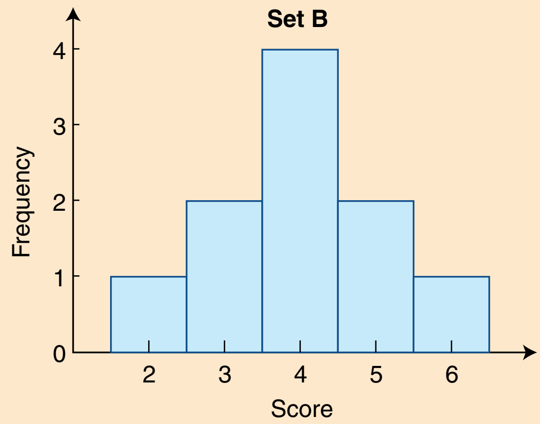
Repeat for the other set of data and you will have two box plots, one on top of the other.

Explain how the box and whisker plots together give a good visual comparison of the sets of data.

Which is a better measure of the spread of data in this case, the range or the inter-quartile range? Why?

2 Consider the following sets of data.





i Describe the spread of the scores from the histograms provided.
Complete the following table using your calculator or otherwise and construct box and whisker plots to represent this information visually.

	Set A	*Set B*
Mean		
Median		
Range		
Q1		
Q2		
IQR		

ii Describe the spread of the scores from the box and whisker plots you constructed.
iii Do you think the range or the inter-quartile are a good measure of the spread of the scores in this case?

Now calculate the standard deviation of each set of scores using your calculator as shown in section 15:01B.

Do you think the standard deviation is a better measure of the spread of these sets of scores? Why?

You might want to refer back to section 15:01B to help with your answer.

3 Suppose the height of an object dropped from a tall building is modelled by the quadratic function $y = 180 - 9{\cdot}8x^2$, where y is the height after x seconds. Find the equation of the inverse of this function and use it to find:

a when the height is 100 m.
b how long the object is in the air.

Assessment Grid for Investigation 15:01 | Comparing sets of data

The following is a sample assessment grid for this investigation. You should carefully read the criteria *before* beginning the investigation so that you know what is required.

Assessment Criteria (C, D) for this investigation				Achieved ✓
Criterion C Communication in Mathematics	a	None of the following descriptors have been achieved.	0	
	b	There is a basic use of mathematical language and representation. Lines of reasoning are insufficient.	1	
			2	
	c	There is satisfactory use of mathematical language and representation. Graphs, tables and explanations are clear but not always logical or complete. Calculations are easy to follow.	3	
			4	
	d	A good use of mathematical language and representation. Graphs and tables are accurate, to scale and fully labeled. Explanations are complete and concise. Mathematical arguments are well written with explicit terminology and support.	5	
			6	
Criterion D Reflection in Mathematics	a	None of the following descriptors have been achieved.	0	
	b	An attempt has been made to explain whether the results make sense and are consistent, and to use them to make comparisons between data sets.	1	
			2	
	c	There is a correct but brief explanation of whether results make sense and how they can be used to make meaningful comparisons between different groups.	3	
			4	
	d	All comparisons of data sets are well written and detailed, showing reflection on the statistical results from analyses. Measures of centre and spread are calculated to an appropriate degree of accuracy and critically compared to each other, considering pros and cons. All statistics are combined used effectively to draw meaning and relevance to the groups being compared.	5	
			6	

15:02 | Using the Standard Deviation

Investigation 15:01 demonstrated that the range and inter-quartile range, although useful in some cases, do not always give a good indication of the spread of the data.

■ The standard deviation is better because it measures the average distance of scores from the mean, or the middle of the data.

To find the standard deviation we first of all work out the distance between each score and the mean:
- Since some of these are negative, each is squared.
- These are now added.
- The mean of these is now calculated.
- Now we need to find the square root as the distances were squared in the first place.

This is the formula for standard deviation.

■ The greater the standard deviation, the more spread out the scores are from the mean.

However, we rely on the calculator to calculate the standard deviation by entering the scores.

worked examples

The last 10 assignment results for Thilo and Sabrina are shown below.

Thilo: 40, 60, 60, 58, 50, 55, 61, 90, 71, 75
Sabrina: 60, 40, 58, 68, 58, 59, 61, 65, 90, 57

a Complete the table:

	Thilo	*Sabrina*
Mean		
Maximum score		
Minimum score		
Standard deviation		

b Whose scores were spread least from the mean?
c Who performed most consistently?

Solution

a

	Thilo	*Sabrina*
Mean	62	61·4
Maximum score	90	90
Minimum score	40	40
Standard deviation	13·2	11·4

b Sabrina, since the standard deviation of her scores is less.
c Sabrina. Her scores are less spread out.

Exercise 15:02

1 Ted and Anna have been keeping a record of the time (in seconds) it takes them to swim one length of the school's 25-metre pool. Their last eight times are shown below:

Ted: 25·0, 26·1, 21·4, 22·8, 24·7, 25·6, 27·0, 28·9

Anna: 28·3, 30·3, 23·2, 26·0, 27·5, 27·9, 28·0, 28·2

a Complete the table

	Ted	*Anna*
Mean		
Maximum time		
Minimum time		
Standard deviation		

b What is the range of scores for each swimmer?

c Who is the most consistent swimmer based on these statistics?

d Explain the last two answers.

2 The school's soccer coach has kept records of the number of shots at goal players have made per game. Below are the results for the team's top two players in the first 14 games of the season:

Mattheus: 1, 1, 3, 2, 3, 2, 5, 4, 6, 7, 4, 5, 6, 7

Emil: 5, 2, 5, 4, 3, 7, 6, 4, 2, 6, 1, 4, 3, 4

a Calculate the mean and standard deviation for each player.

b Which player is more likely to have close to 4 shots at goal in the next game?

c If the coach had to pick just one of these two players, why might he pick Mattheus? What allows him to make this decision?

3 Two mini marts on opposite sides of the street record how many customers they have per hour over a 10-day period. The results are shown below:

Max's mini-mart: 5, 6, 5, 8, 9, 8, 7, 6, 7, 9

Sam's mini-mart: 7, 8, 5, 6, 7, 8, 9, 6, 7, 7

a Calculate the mean and inter-quartile range for each mini-mart.

b Is it possible to tell from these statistics which mini-mart has the most regular number of customers? Why?

c Use the standard deviation to decide which has the most regular number of customers.

4 Hillary and Bron are comparing their end-of-semester reports They score grades out of 7 in each of their six subjects (a grade of 7 is the highest). Their scores are shown below.

	Hillary	*Bron*
English	4	3
Mathematics	5	4
History	3	4
IT	4	5
Science	3	6
Mandarin	7	4

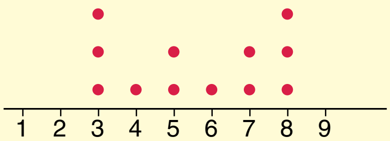
a Calculate their grade point average (the mean of their grades).

b By looking at their scores, who do you think is most consistent between subjects?

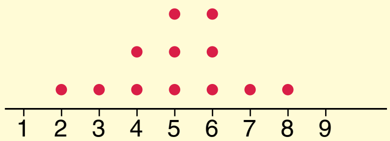
c Show this using standard deviation.

5 Huy has two places where he sets traps to catch crabs. To find out which produces more consistent results, he graphed the results of each place over 12 days. The results are shown in the dot plots below. Each day Huy put a dot above the number of crabs in each trap.

In the estuary



Off the beach

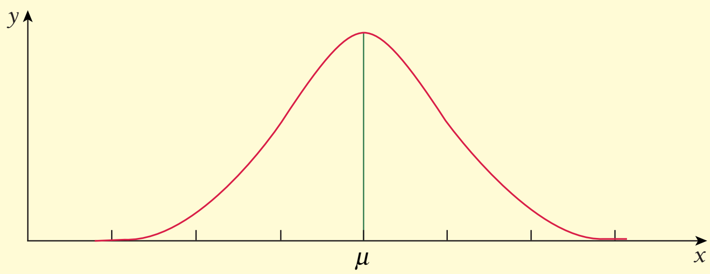


a Calculate the mean and standard deviation for each place.
b Which place gives the most consistent results?
c Why might Huy choose to use the other place regardless of the statistics?

15:03 | The Normal Distribution

Usually, when a large sample is taken and the results graphed, a *normal* or *bell-shaped* curve is obtained.

The normal distribution



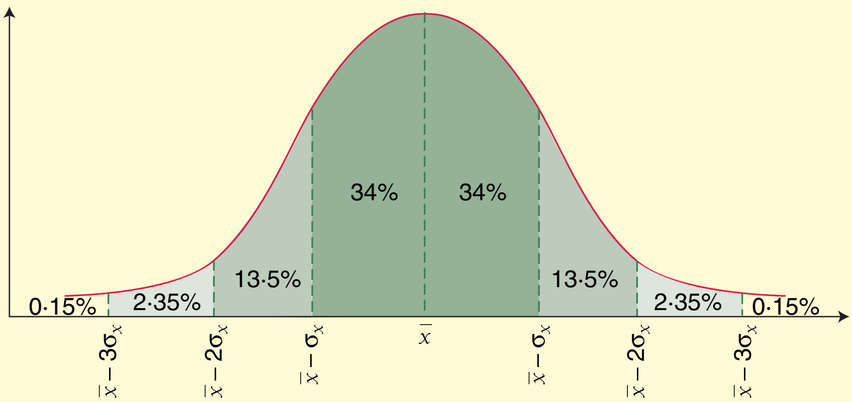
Examples of statistics which might produce a normal curve come from populations or samples:
- The mass of Grade 10 boys in the country
- The number of matches in a box produced by a factory
- The length of 10 cm nails produced by a machine
- The results of a statewide spelling test.

Although some will be less than the mean and some will be more than the mean, the distribution of the population should be symmetrical about the mean.

> Definition: In a normal distribution:
> Approximately 68% of the scores lie within one standard deviation of the mean.
> Approximately 95% of the scores lie within two standard deviations of the mean.
> Approximately 99·7% of the scores lie within three standard deviations of the mean.

This suggests that the standard deviation is important when using, measuring and defining a normal distribution.

Percent of normal distribution scores in each interval



worked examples

A machine is used to fill 2-litre tins with paint. It has been found that the amount of paint in the tins has a mean of 2 litres, with a standard deviation of 10 mL.

Approximately what proportion of the tins contain

a between 1·990 litres and 2·010 litres of paint?

b between 1·980 litres and 2·020 litres of paint?

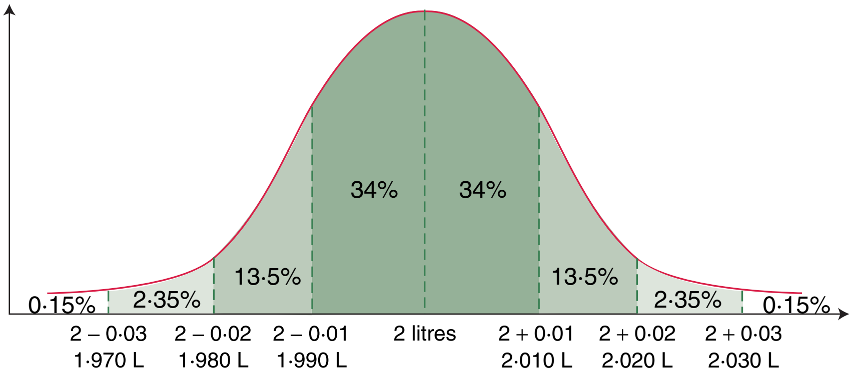
c more than 2·010 litres of paint?

d less than 1·970 litres of paint?

continued →→→

Solution

Using the diagram of a normal distribution on which the percentages are marked:



a between 1·990 litres and 2·010 litres = 34% + 34% = 68% of the tins

b between 1·980 litres and 2·020 litres = 13·5% + 34% + 34% + 13·5% = 95% of tins

c more than 2·010 litres = 100% – (50% + 34%) = 16%

d less than 1·970 litres = 0·15%

When considering the probability of choosing a score at random, the following terminology is usually used.

- If a score is chosen at random from a normal distribution:
  It will *most probably* lie within one standard deviation of the mean.
  It will be *very likely* that it lies within two standard deviations of the mean.
  It will *almost certainly* lie within three standard deviations of the mean.

Exercise 15:03

1 A machine is producing 5 cm screws. It is found that the screws it produces are in normal distribution with a mean of 5 cm and a standard deviation of 0.8 mm.

i Approximately what percentage of the screws produced have a length:

a between 4·84 cm and 5·16 cm?

b between 47·6 mm and 50 mm?

c greater than 50·8 mm?

ii All screws with length greater than 51·6 mm are rejected. What percentage of screws are rejected?

2 In a Grade 10 test, the results were in normal distribution with a mean score is 60% with a standard deviation of 12·5%.

i What proportion of Grade 10 students scored between

a 60% and 85%?

b 22·5% and 35%?

c 72·5% and 85%?

ii If 400 students sat the test and Jeremy scored 85%, how many students scored higher than Jeremy?

3 The heights of one-month-old saplings in a new park form a normal distribution. It is known that the middle 68% of the saplings have heights between 0·75 metres and 1·15 metres.

- **a** What is the mean height?
- **b** What is the approximate maximum height of a sapling?
- **c** What is the approximate minimum height of a sapling?
- **d** What is the minimum height of the tallest 2·5% of saplings?

4 The fully charged battery life of a new brand of laptop computer is in normal distribution with a mean life of 2 hours. It is found that only 2·5% of the computers have a battery life of longer than 2 hours and 30 minutes.

- **a** What is the standard deviation?
- **b** What is the approximate maximum battery life?
- **c** What is the approximate minimum battery life?
- **d** If a computer was selected at random, between what two times would the battery life almost certainly lie?

5 A machine produces metal rods, the length of which forms a normal distribution with a mean length of 85 cm. It is found that out of every 500 metal rods that are produced, 170 are between 85 cm and 88 cm long.

- **a** What is the standard deviation of the length of the rods?
- **b** How many rods have a length greater than 88 cm?
- **c** If a rod is chosen at random, between what two lengths will it most probably lie?
- **d** What proportion of the rods have a length under 82 cm?

15:04 | Statistics with Two Variables

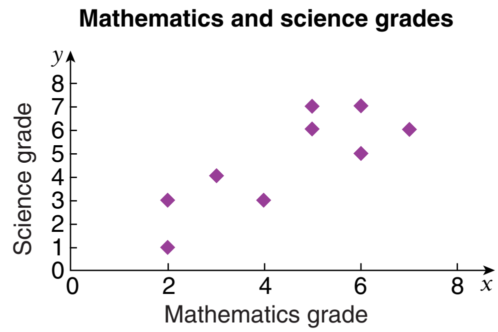
Often data is collected which contains two types of data that may, or may not be related. This is known as **bivariate data**.

worked examples

1 The table below shows the mathematics and science grades for a group of Grade 10 students.

Mathematics	2	7	6	5	4	5	5	3	2	6
Science	1	6	7	6	3	7	6	4	3	5

When graphed against each other, a series of points is obtained as shown. This type of graph is called a **scatter graph** or a **scatter plot**.



It can be seen that there seems to be a relationship between the mathematics and science grades. As the mathematics grades get higher, so do the science grades.

We call such a relationship a *correlation* between variables.

If a student does well in mathematics, it seems he/she also does well in science.

A relationship such as this is called a *positive correlation*.

2 The scatter graph shows the percentage of car accidents for drivers of a given age in a particular town.

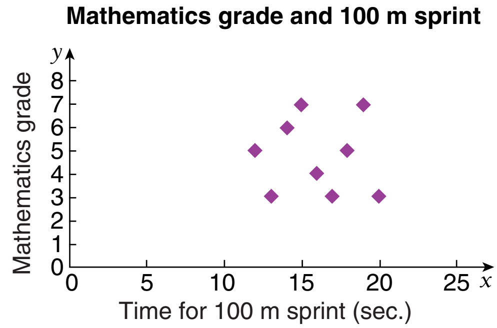
It seems that here there is also a relationship between the variables. However, as one variable increases, the other decreases.

A relationship such as this is called a *negative correlation*.

- A point for discussion: Do you think this trend would continue? Why or why not?

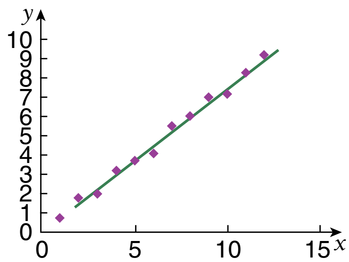
3 This scatter graph compares students' times over a 100 m sprint and their mathematics grade. As perhaps you would expect, there is no relationship between these variables.

When there is no relationship and points are randomly scattered, there is *no correlation*.

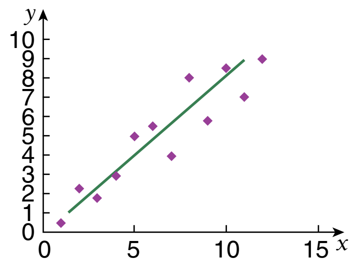


As well as describing whether the correlation is negative or positive, we can describe how strong the correlation is.

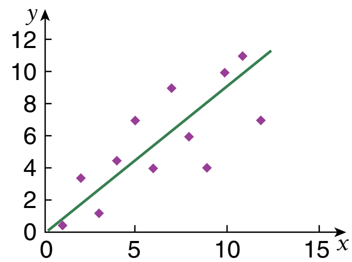
For example:



Here there is a ***strong*** *positive correlation.*



Here there is a ***moderate*** *positive correlation.*



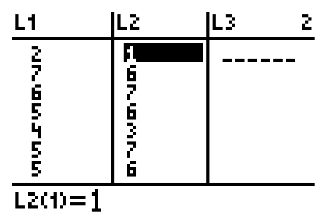
Here there is a ***weak*** *positive correlation.*

Not only are these correlations positive but they are also *linear* — the points seem to form a straight line.

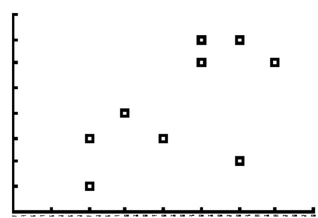
Using a calculator to describe correlation and to find the line of best fit

Consider the data in worked example 1.

This can be entered into the calculator using the STAT function.



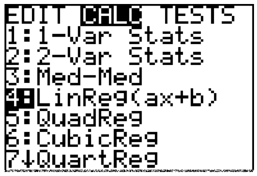
We can then graph these values as a scatter graph.
To do this use the STAT PLOT function.
Turn on only one Stat Plot, choose the scatter graph and make sure you enter the correct lists for the X and Y axes.

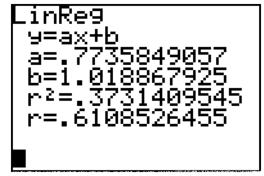


If the data seems to be linear, your calculator will also find the line that best fits the data. This is called the **line of best fit**.

continued →→→

To find the equstion of the line we perform a *linear regression* on the data.

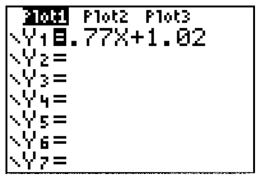




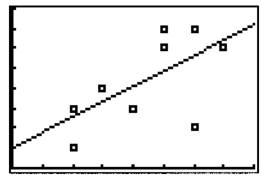
Note: you must have the *diagnostic* turned on. To do this got to *catalog/diagnostic/on*.

This tells us that the gradient of the line is 0·774 and the y-intercept is 0·611.

You can now graph the line



Enter the equation

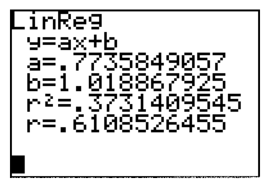


Graph

As well as using the graph to describe the correlation as strong, moderate and weak, we can use the *correlation coefficient* to help describe the strength of the correlation.

The correlation coefficient ranges from −1 (a perfect negative correlation) to 1 (a perfect positive correlation).

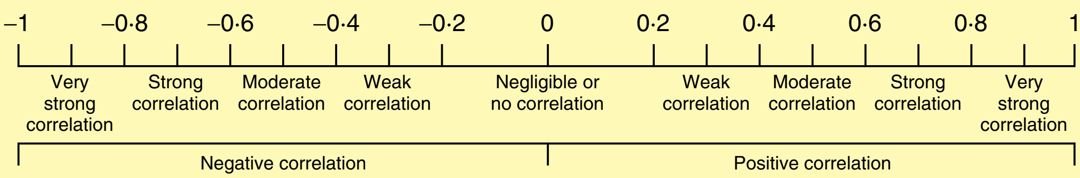
The TI gives a *correlation coefficient* when the linear regression is done.



It is given by the *r* value.

In this case $r = 0{\cdot}61$.

To interpret the correlation coefficient, the following guideline can be used:



So, in the example above, it could be said that there is a moderate to strong positive correlation between mathematics grades and science grades.

Exercise 15:04

1 From the following sets of data, determine the correlation coefficient and describe the type and strength of the correlation using the information above.

a The table shows the number of days of rain in the months of a particular year and the number of student absences in the same months in a large international school.

	Jan	*Feb*	*Mar*	*Apr*	*May*	*June*	*July*	*Aug*	*Sept*	*Oct*	*Nov*	*Dec*
Days of rain in the month	5	4	1	0	8	17	15	20	18	11	6	3
Student absences from school	25	22	7	5	46	89	85	100	95	60	30	22

b The table shows the history and mathematics marks scored by 10 students in recent tests.

History mark	50	65	82	98	43	20	68	72	75	69
Mathematics mark	88	90	64	70	60	45	90	65	78	95

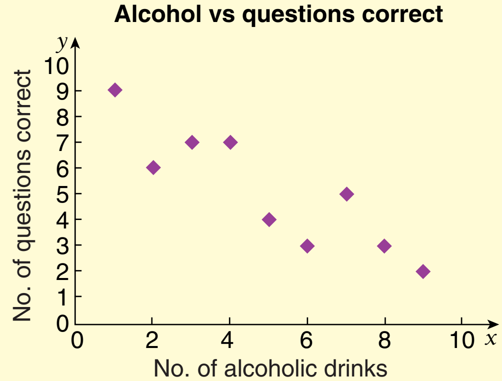
c The table shows the number of hours students spent playing computer games and the average grade they scored in their semester report.

Hours on computer per week	*Average grade*
0	6
2	4
14	3
17	3
10	4
5	6
3	4
18	3
8	4
11	4

d The table shows the percentage capacity at a football ground when games are played in different temperatures.

Temperature (°C)	5	−5	8	12	−1	10	−8	9	7	8
Capacity (%)	75	80	85	60	78	85	80	100	88	90

e The graph shows the results of an experiment in which 10 subjects were asked to drink a prescribed number of alcoholic drinks and complete a multiple choice IQ test with 10 questions.



2 For each of those examples in question 1, find the equation of the line of best fit. Use 3 significant figures.

3 The table shows the yield in tonnes of produce on farms when sprayed with different amounts of insecticide concentrate.

Insecticide (mL)	5	10	15	20	25	30
Yield (tonnes)	220	380	400	320	360	480

a Find the correlation coefficient.
b Describe the correlation between the variables.
c Find the equation of the line of best fit (use 3 significant figures).
d Use the line of best fit to predict the yield if 35 mL of insecticide was used.
e Use the line of best fit to estimate the yield if no insecticide was used.

4 On the same farm, samples of pests were taken and numbers recorded below:

Insecticide (mL)	5	10	15	20	25	30
No of pests per plant	62	52	28	15	10	9

a Find the correlation coefficient.
b Describe the correlation between the variables.
c Find the equation of the line of best fit (use 3 significant figures).
d Use the line of best fit to predict the number of pests if 35 mL of insecticide was used.
e Explain your answer in **d**.

5 The table shows the amount of fuel left in the tank of a car during a trip, compared to the time travelled. The car started the trip with a full tank of fuel.

Length of trip (minutes)	30	90	180	210	270	315
Fuel in tank (litres)	55	32	30	15	14	5

a Find the correlation coefficient.
b Describe the correlation in words.
c Find the equation of the line of best fit (use 3 significant figures).
d Use the line of best fit to approximate how much fuel the tank holds when full.
e Approximate how long the car will last on a tank of fuel if the same pattern of driving continues.
f Give possible reasons why the rate of fuel consumption changes throughout the trip.

Mathematical Terms 15

bivariate data
- data collected that has two variables.

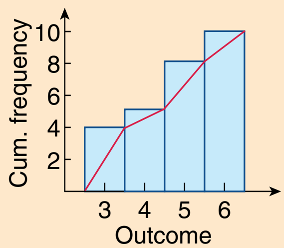
class interval
- The size of the groups into which the data is organised.
  eg 1–5 (5 scores); 11–20 (10 scores).

class centre
- The middle outcome of a class.
  eg The class 1–5 has a class centre of 3.

correlation
- a connection between sets of data. This can be negative or positive, linear or non linear.

cumulative frequency
- The number of scores less than or equal to a particular outcome.
  eg For the data 3, 6, 5, 3, 5, 5, 4, 3, 3, 6 the cumulative frequency of 5 is 8 (there are 8 scores of 5 or less).

cumulative frequency histogram (and polygon)
- These show the outcomes and their cumulative frequencies.



frequency
- The number of times an outcome occurs in the data.
  eg For the data 3, 6, 5, 3, 5, 5, 4, 3, 3, 6 the outcome 5 has a frequency of 3.

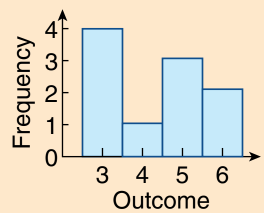
frequency distribution table
- A table that shows all the possible outcomes and their frequencies. (It usually is extended by adding other columns such as the cumulative frequency.)

eg

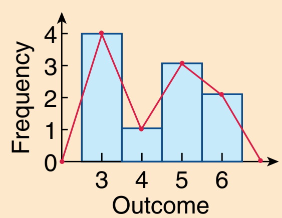
Outcome	*Frequency*	*Cumulative frequency*
3	4	4
4	1	5
5	3	8
6	2	10

frequency histogram
- A type of column graph showing the outcomes and their frequencies.

eg



frequency polygon
- A line graph formed by joining the midpoints of the top of each column. To complete the polygon the outcomes immediately above and below those present are used. The heights of these columns is zero.



grouped data
- The organisation of data into groups or classes.

inter-quartile range
- 3rd quartile – 1st quartile

line of best fit
- the line that best fits the data when graphed

mean
- The number obtained by 'evening out' all the scores until they are equal.
  eg If the scores 3, 6, 5, 3, 5, 5, 4, 3, 3, 6 were 'evened out' the number obtained would be 4·3.
- To obtain the mean use the formula:

$$\text{Mean} = \frac{\text{sum of scores}}{\text{total number of scores}}$$

median
- The middle score for an odd number of scores or the mean of the middle two scores for an even number of scores.

mode (modal class)
- The outcome or class that contains the most scores.

median class
- In grouped data the class that contains the median.

normal distribution
- when the data forms a bell shaped curve in which:
  – Approximately 68% of the scores lie within one standard deviation of the mean.
  – Approximately 95% of the scores lie within two standard deviations of the mean.
  – Approximately 99.7% of the scores lie within three standard deviations of the mean.

ogive
- This is another name for the cumulative frequency polygon.

outcome
- A possible value of the data.

range
- The difference between the highest and lowest scores.

range
- highest score – lowest score

standard deviation (σ_n)
- average distance of the scores from the mean.

statistics
- The collection, organisation and interpretation of numerical data.

Technology Applications Drag and Drops

Mathematical terms 15

Chapter 15 | Revision Assignment

assignment

15A

1 The data below gives the average monthly minimum daily temperatures of two Australian cities. The months are in order: January to December.

Adelaide: 15·5, 15·7, 14·3, 11·6, 9·4, 7·4, 6·8, 7·5, 8·6, 10·4, 12·3, 14·3

Alice Springs: 21·2, 20·6, 17·4, 12·5, 8·2, 5·1, 4, 5·9, 9·7, 14·8, 17·9, 20·2

- a Calculate the mean and standard deviation for each city.
- b Construct grouped frequency distribution tables with the classes 0–<5, etc. for both cities' distributions.
- c Draw a cumulative frequency polygon of the each distribution to estimate:
  - i the median
  - ii Q1
  - iii Q3
  - iv the interquartile range
- d Construct a box and whisker plot of each cities' distributions.
- e Compare the measures of centre for each distribution.
- f Discuss the spread of each distribution.
- g In which city would you rather live? Why?

2 Bags of cement are labelled 25 kg. The bags are filled by machine and the actual weights are normally distributed with mean 25·5 kg. It is known that 16% of bags have less than 25 kg.

- a What is the standard deviation?
- b What is the approximate maximum weight of a bag?
- c What is the approximate minimum weight of a bag?
- d What proportion of the bags weigh between 24 and 26 kg?

3 It is decided to take a random sample of 10 students to see if there is any linear relationship between height and shoe size. The results are given in the table below.

Height (cm)	*Shoe size*
175	8
160	9
180	8
155	7
178	10
159	8
166	9
185	11
189	10
173	9

- a Find the correlation coefficient.
- b Describe the correlation between height and shoe size.
- c Find the equation of the line of best fit.
- d Predict the shoe size of a student who is 162 cm in height.
- e Predict the height of someone with a shoe size of 13.
- f Why is your answer for **e** less reliable than your answer for **d**?