YEAR 10 EXTENDED.

Name: Answers. Date: ____________

Test: Sequences, Series and Financial Math

1. Using an appropriate formula for t_n, calculate the 12th term of the sequence { 5, 10, 20, 40, ... }.

$\times 2 \; \times 2 \; \times 2$

Geometric Sequence.

$t_n = ar^{n-1}$

$a = 5$

$r = 2$

$n = 12$

$t_{12} = 5(2^{11})$

$= 10240$ //.

2. Using an appropriate formula for t_n, calculate the 45th term of the sequence { -90, -83, -76, -69, ... }

$+7 \; +7 \; +7$.

Arithmetic Sequence.

$t_n = a + (n-1)d$

$a = -90$

$d = 7$.

$n = 45$

$t_{45} = (-90) + (45-1)(7)$

$= 218$ //

3. The number 546 occurs as a term in the sequence { -65, -52, -39, -26, ... }. Which term is it? (Put another way, for what n does $t_n = 546$?) Use an appropriate t_n formula.

Arithmetic Sequence.

$t_n = 546$.

$a = -65$

$d = +13$.

$t_n = a + (n-1)d$.

$546 = (-65) + (n-1)(13)$.

$546 = -65 + 13n - 13$.

$546 + 78 = 13n$.

$624 = 13n$.

$n = 48$ //

$\therefore 48^{th}$ term.

$\times 3 \quad \times 3 \quad \times 3$

4. The number 6561 occurs as a term in the sequence { $\frac{1}{9}$, $\frac{1}{3}$, 1, 3 ... }. Which term is it? (Put another way, for what n does t_n = 6561?) Use an proper t_n formula.

Geometric Sequence.

$t_n = 6561$

$r = 3$

$a = \frac{1}{9}$

$t_n = ar^{n-1}$

$6561 = \frac{1}{9}(3^{n-1})$

$59049 = 3^{n-1}$

$\log 59049 = (n-1)\log 3$

$n-1 = \frac{\log 59049}{\log 3}$

$n-1 = 10$

$n = 11 \therefore$ 11th term!

5. Find the general term (t_n = ...) for an arithmetic sequence where t_9 = 30 and t_{22} = 108.

$\left.\begin{array}{l} t_9 = 30, n = 9 \\ t_{22} = 108, n = 22 \end{array}\right\}$ find a, d. $\Rightarrow t_n = a + (n-1)d$

$t_9 = a + (9-1)d$

$30 = a + (9-1)d$

① $30 = a + 8d$

$t_{22} = a + (22-1)d$

② $108 = a + 21d$

$\begin{cases} 30 = a + 8d & ① \\ 108 = a + 21d & ② \end{cases}$

① - ②

$-78 = -13d$

$d = 6$

$30 = a + 8(6)$

$30 = a + 48$

$a = -18$

$\therefore t_n = -18 + (n-1)(6)$

$\boxed{t_n = 6n - 24}$ //

6. Find the general term (t_n = ...) for an geometric sequence where t_7 = 5.76 and t_{14} = 737.28.

$\left.\begin{array}{l} t_7 = 5.76, n = 7 \\ t_{14} = 737.28, n = 14 \end{array}\right\}$ find a, r. $\Rightarrow t_n = ar^{n-1}$

$\begin{cases} 5.76 = ar^6 & ① \\ 737.28 = ar^{13} & ② \end{cases}$

from ①

$a = \frac{5.76}{r^6}$, sub into ②

$737.28 = \left(\frac{5.76}{r^6}\right) r^{13}$

$737.28 = 5.76 r^7$

$r^7 = 128$

$r = 2$

$\therefore a = \frac{5.76}{2^6} = \frac{9}{100} = 0.09$

$\therefore t_n = (0.09)(2^{n-1})$

$t_n = \left(\frac{9}{100}\right)(2^{n-1})$ //

or.

or $t_n = \left(\frac{9}{200}\right)(2^n)$ //

7. Using an appropriate formula for S_n, calculate the sum of: $-64 - 57 - 50 - 43 - \ldots + 90 + 97$

$+7 \quad +7 \quad +7$

Arithmetic Series

$S_n = \frac{n}{2}[2a + (n-1)d]$

① Find n when $t_n = 97$

$97 = (-64) + (n-1)(7)$

$97 = -64 + 7n - 7$

$7n = 168$

$n = 24$

②

$S_{24} = \frac{24}{2}[2(-64) + (23)(7)]$

$= 12(-128 + 161)$

$= 396$

8. Using an appropriate formula for Sn, calculate the sum of: $1024 + 512 + 256 + \ldots + \frac{1}{8} + \frac{1}{16}$

$\times\frac{1}{2} \quad \times\frac{1}{2}$

Geometric Series

$S_n = \frac{a(r^n - 1)}{r-1}$ $\quad r = \frac{1}{2}$ $\quad a = 1024$

① Find n when $t_n = \frac{1}{16}$

$t_n = ar^{n-1}$

$\frac{1}{16} = 1024\left(\frac{1}{2}\right)^{n-1}$

$\frac{1}{16384} = \frac{1}{2}^{n-1}$

$\log\frac{1}{16384} = (n-1)\log\frac{1}{2}$

$n - 1 = \frac{\log\frac{1}{16384}}{\log\frac{1}{2}}$

$n - 1 = 14$

$n = 15$

② Find Sn

$S_{15} = \frac{1024\left(\frac{1}{2}^n - 1\right)}{\frac{1}{2} - 1}$

$= \frac{1024\left(\frac{1}{2}\right)^{15} - 1024}{-\frac{1}{2}}$

$= 2047.9375 = 2047\frac{15}{16}$

or $\frac{32767}{16}$

$$S_{15} = \frac{1024\left(\frac{1}{2}^{15} - 1\right)}{\frac{1}{2} - 1} = \frac{1024\left(\frac{1}{32768} - 1\right)}{-\frac{1}{2}} = \frac{1024\left(-\frac{32767}{32768}\right)}{-\frac{1}{2}}$$

$$= \left(-\frac{32767}{32768}\right)(-2048)$$

$$= \frac{32767}{16}$$

$$= 2047\frac{15}{16}$$

Arithmetic Sequence: $t_n = a + (n-1)d$

Geometric Sequence: $t_n = ar^{n-1}$

Arithmetic Series: $S_n = \frac{n}{2}(a + t_n)$ $= S_n = \frac{n}{2}(a + a + (n-1)d)$.

Geometric Series: $S_n = \frac{a(r^n - 1)}{r - 1}$ $S_n = \frac{n}{2}\left[2a + (n-1)d\right]$