Y11 MATHEMATICAL STUDIES – SAW REVIEW 2012

1. A marine biologist records as a frequency distribution the lengths (L), measured to the nearest centimetre, of 100 mackerel. The results are given in the table below.

Length of mackerel (L cm)	Number of mackerel
$27 < L \leq 29$	2
$29 < L \leq 31$	4
$31 < L \leq 33$	8
$33 < L \leq 35$	21
$35 < L \leq 37$	30
$37 < L \leq 39$	18
$39 < L \leq 41$	12
$41 < L \leq 43$	5
	100

(a) Construct a cumulative frequency table for the data in the table.

(2)

(b) Draw a cumulative frequency curve.

Hint: Plot your cumulative frequencies at the top of each interval.

(3)

(c) Use the cumulative frequency curve to find an estimate, to the nearest cm for

(i) the median length of mackerel;

(2)

(ii) the interquartile range of mackerel length.

(2)
(Total 9 marks)

2. The following table shows the times, to the nearest minute, taken by 100 students to complete a mathematics task.

Time (t) minutes	11–15	16–20	21–25	26–30	31–35	36–40

Number of students	7	13	25	28	20	7

(a) Construct a cumulative frequency table. (Use upper class boundaries 15.5, 20.5 and so on.)

(2)

(b) On graph paper, draw a cumulative frequency graph, using a scale of 2 cm to represent 5 minutes on the horizontal axis and 1 cm to represent 10 students on the vertical axis.

(3)

(c) Use your graph to estimate

(i) the number of students that completed the task in less than 17.5 minutes;

(ii) the time it will take for $\frac{3}{4}$ of the students to complete the task.

(2)
(Total 7 marks)

3. For the set of {8, 4, 2, 10, 2, 5, 9, 12, 2, 6}

(a) calculate the mean;

(b) find the mode;

(c) find the median.

Working:

Answers:

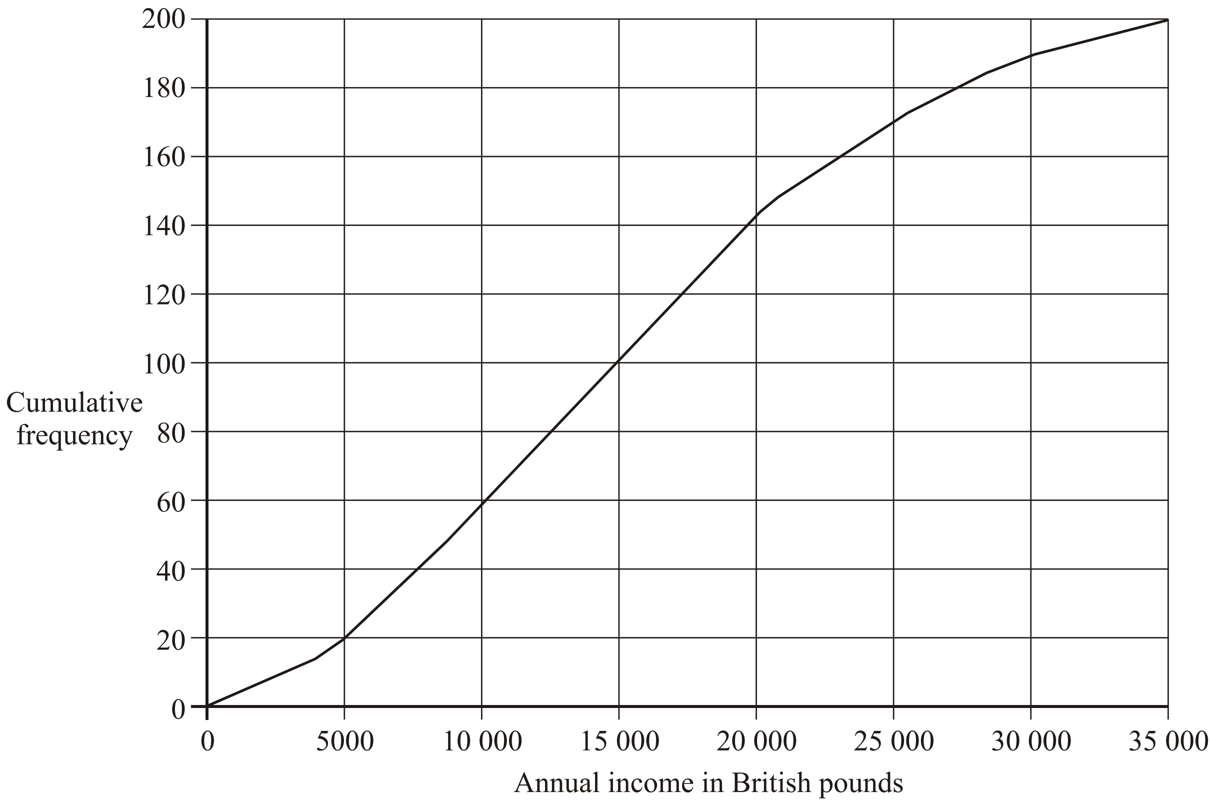
(a) …………………………………………..

(b) ....

(c) ……………………………………..........

(Total 4 marks)

4. The graph below shows the cumulative frequency for the yearly incomes of 200 people.



Use the graph to estimate

(a) the number of people who earn less than 5000 British pounds per year;

(b) the median salary of the group of 200 people;

(c) the lowest income of the richest 20% of this group.

Working:

Answers:

(a) ....

(b) ....

(c) ....

(Total 4 marks)

5. The weight in kilograms of 12 students in a class are as follows.

63 76 99 65 63 51 52 95 63 71 65 83

(a) State the mode.

(1)

(b) Calculate

(i) the mean weight;

(ii) the standard deviation of the weights.

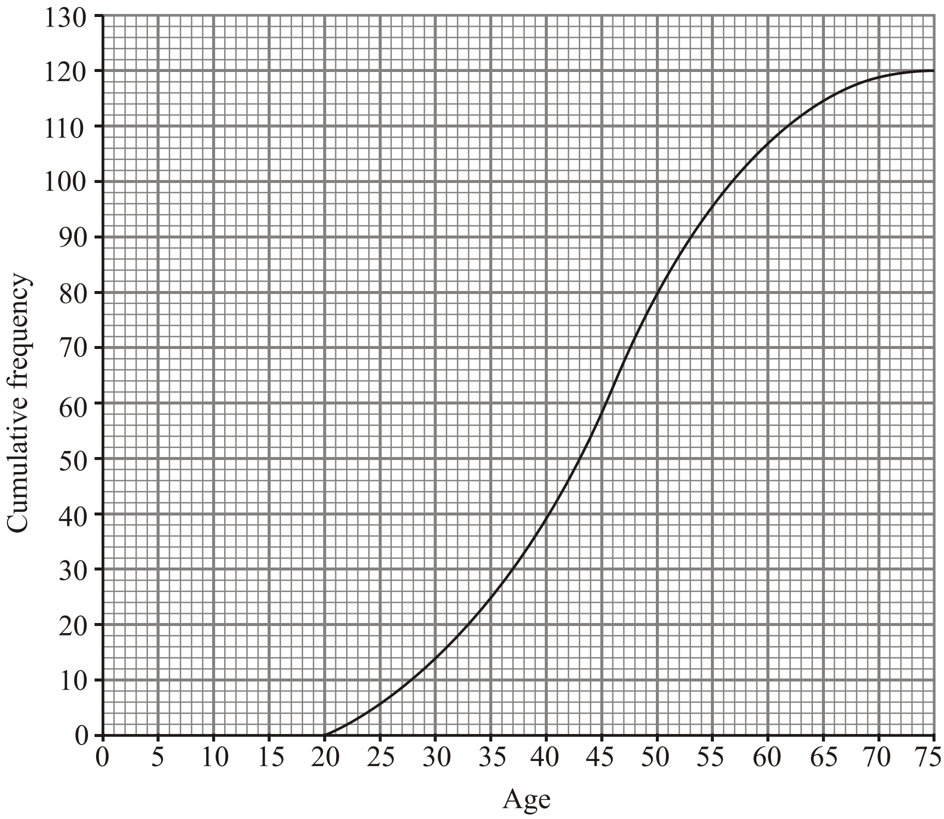
(2)

When one student leaves the class, the mean weight of the remaining 11 students becomes 70 kg.

(c) Find the weight of the student who left.

(2)
(Total 5 marks)

6. There are 120 teachers in a school. Their ages are represented by the cumulative frequency graph below.



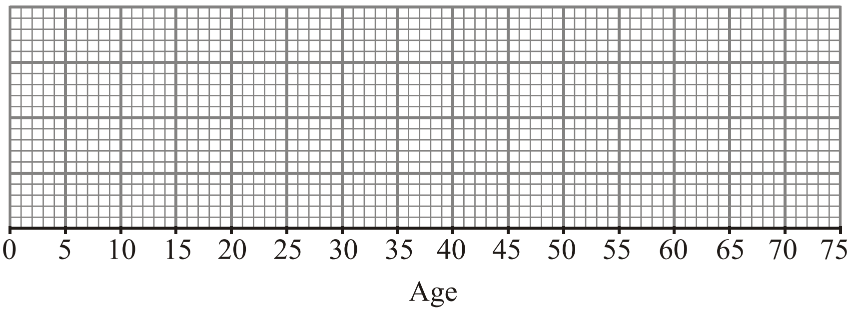
(a) Write down the median age.

(1)

(b) Find the interquartile range for the ages.

(2)

(c) Given that the youngest teacher is 21 years old and the oldest is 72 years old, represent the information on a box and whisker plot using the scale below.



(3)

Working:

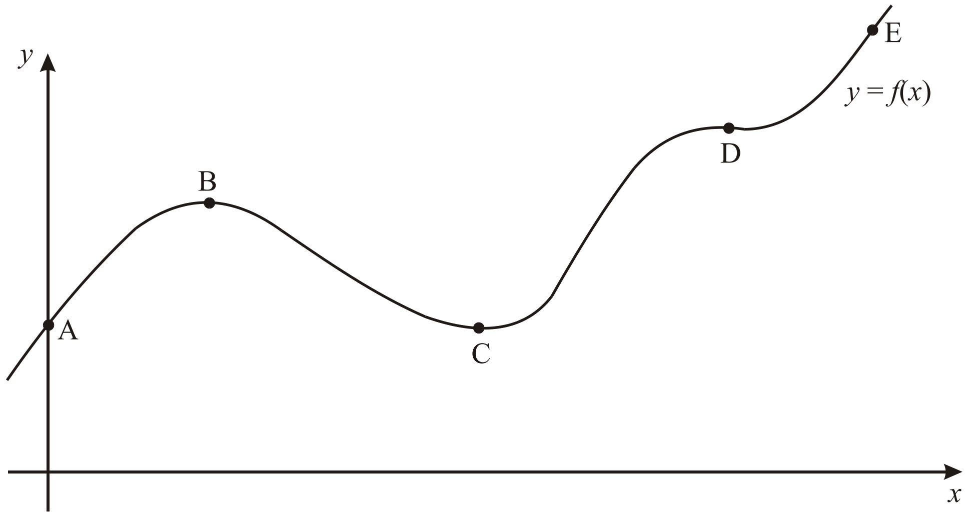
Answers:

(a) ....

(b) ....

(Total 6 marks)

7. A, B, C, D and E are points on the curve $y = f(x)$ shown in the diagram below.



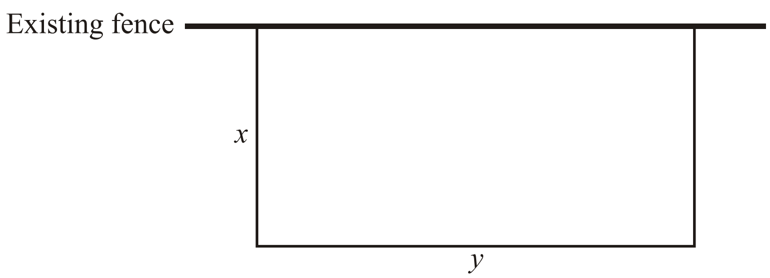
(a) Describe the gradient of the curve in passing from the point B, through point C to point D.

(3)

(b) D has coordinates $(a, f(a))$, and the x-coordinate at E is $a + 4$. Write an expression for the gradient of the line segment [DE].

(3)
(Total 6 marks)

8. A farmer wishes to enclose a rectangular field using an existing fence for one of the four sides.



(a) Write an expression in terms of x and y that shows the total length of the new fence.

(1)

(b) The farmer has enough materials for 2500 metres of new fence. Show that

$$y = 2500 - 2x$$

(1)

(c) $A(x)$ represents the area of the field in terms of x.

(i) Show that

$$A(x) = 2500x - 2x^2$$

(2)

(ii) Find $A'(x)$.

(1)

(iii) Hence or otherwise find the value of x that produces the maximum area of the field.

(3)

(iv) Find the maximum area of the field.

(3)
(Total 11 marks)

9. At the circus a clown is swinging from an elastic rope. A student decides to investigate the motion of the clown. The results can be shown on the graph of the function $f(x) = (0.8^x)(5 \sin 100x)$, where x is the horizontal distance in metres.

(a) Sketch the graph of $f(x)$ for $0 \le x \le 10$ and $-3 \le f(x) \le 5$.

(5)

(b) Find the coordinates of the first local maximum point.

(2)

(c) Find the coordinates of one point where the curve cuts the y-axis.

(1)

Another clown is fired from a cannon. The clown passes through the points given in the table below:

Horizontal distance (x)	Vertical distance (y)
0.00341	0.0102
0.0238	0.0714
0.563	1.69
1.92	5.76
3.40	10.2

10. (a) On the same graph sketch the curves $y = x^2$ and $y = 3 - \frac{1}{x}$ for values of x from 0 to 4 and values of y from 0 to 4. Show your scales on your axes.

(4)

(b) Find the points of intersection of these two curves.

(4)

(c) (i) Find the gradient of the curve $y = 3 - \frac{1}{x}$ in terms of x.

(ii) Find the value of this gradient at the point (1, 2).

(4)

(d) Find the equation of the tangent to the curve $y = 3 - \frac{1}{x}$ at the point (1, 2).

(3)
(Total 15 marks)

11. The function $f(x)$ is given by $f(x) = x^3 - 3x^2 + 3x$, for $-1 \le x \le 3$.

(a) Differentiate $f(x)$ with respect to x.

(2)

(b) Complete the table below.

x	-1	0	1	2	3
$f(x)$		0	1	2	9
$f'(x)$	12		0		12

(3)

(c) Use the information in your table to sketch the graph of $f(x)$.

(2)

(d) Write down the gradient of the tangent to the curve at the point (3, 9).

(1)
(Total 8 marks)

12. (a) Write $\frac{3}{x^2}$ in the form $3x^a$ where $a \in \mathbb{Z}$.

(b) Hence differentiate $y = \frac{3}{x^2}$ giving your answer in the form $\frac{b}{x^c}$ where $c \in \mathbb{Z}^+$.

(Total 6 marks)

13. Consider the function $f(x) = x^3 + 7x^2 - 5x + 4$.

(a) Differentiate $f(x)$ with respect to x.

(3)

(b) Calculate $f'(x)$ when $x = 1$.

(2)

(c) Calculate the values of x when $f'(x) = 0$.

(3)

(d) Calculate the coordinates of the local maximum and the local minimum points.

(2)

(e) On graph paper, taking axes $-6 \leq x \leq 3$ and $0 \leq y \leq 80$, draw the graph of $f(x)$ indicating clearly the local maximum, local minimum and y-intercept.

(4)
(Total 14 marks)

14. The velocity, $v\text{ms}^{-1}$, of a kite, after t seconds, is given by

$$v = t^3 - 4t^2 + 4t, \qquad 0 \leq t \leq 4.$$

(a) What is the velocity of the kite after

(i) one second?

(ii) half a second?

(2)

(b) Calculate the values of a and b in the table below.

t	0	0.5	1	1.5	2	2.5	3	3.5	4
v	0			a	0	0.625	b	7.88	16

(2)

(c) (i) Find $\frac{dv}{dt}$ in terms of t. Find the value of t at the local maximum and minimum values of the function.

(ii) Explain what is happening to the function at its local maximum point. Write down the gradient of the tangent to its curve at this point.

(8)

(d) On graph paper, draw the graph of the function $v = t^3 - 4t^2 + 4t$, $0 \leq t \leq 4$. Use a scale of 2 cm to represent 1 second on the horizontal axis and 2 cm to represent 2 ms^{-1} on the vertical axis.

(5)

(e) Describe the motion of the kite at different times during the first 4 seconds.
Write down the intervals corresponding to changes in motion.

(3)
(Total 20 marks)

15. Consider the function $f(x) = 2x^3 - 3x^2 - 12x + 5$.

(a) (i) Find $f'(x)$.

(ii) Find the gradient of the curve $f(x)$ when $x = 3$.

(4)

(b) Find the x-coordinates of the points on the curve where the gradient is equal to -12.

(3)

(c) (i) Calculate the x-coordinates of the local maximum and minimum points.

(ii) Hence find the coordinates of the local minimum.

(6)

(d) For what values of x is the value of $f(x)$ increasing?

(2)
(Total 15 marks)