Build - up Exercise 2A

Fundamental Question

Solve the following equations. (**1 – 8**)

1. $x(2x-3)=0$

2. $(3x+5)x=0$

3. $(x+3)(x-3)=0$

4. $(x-4)(x+2)=0$

5. $(3x+4)(2x-7)=0$

6. $(4x+9)(5x-8)=0$

7. $(x+8)^2=0$

8. $(5x-2)^2=0$

Consolidation Question

Solve the following equations by the factor method. (**9 – 27**)

9. $3x^2-12x=0$

10. $2x^2=10x$

11. $x^2+12x+27=0$

12. $x^2+12x+36=0$

13. $x^2-13x+40=0$

14. $x^2-19x+90=0$

15. $x^2+x-42=0$

16. $x^2+5x-84=0$

17. $x^2-6x-91=0$

18. $x^2-10x-75=0$

19. $6x^2-11x-10=0$

20. $12x^2+37x+21=0$

21. $30x^2-37x+10=0$

22. $24x^2+54x+12=0$

23. $70x^2+32x-6=0$

24. $3x^2-48=0$

25. $(x+3)^2 = 25$

26. $x + 63 = 20x^2$

27. $3(x^2 + 10) = -23x$

Challenging Question

Solve the following equations by the factor method. (**28 – 36**)

28. $(2x+3)^2 - 81 = 0$

29. $64 - (3x-1)^2 = 0$

30. $4(4x-5)^2 - 36 = 0$

31. $(x+7)(x+9) = 3$

32. $(3x-7)(3x-5) = 8$

33. $x(3x-2) - 2x(x-4) = 0$

34. $5(5-2x) = 2x(5-2x)$

35. $(4x+1)(3x-1) - (2x+3)(1-3x) = 0$

36. $(8x+3)^2 = (3x-7)(8x+3)$

37. Solve the equation $x^2 - 2px + p^2 - 16 = 0$, where p is a constant. Express the answers in terms of p.

Build - up Exercise 2B

Fundamental Question

Solve the following equations by the method of taking square roots. (Express the answers in surd form if necessary.) (**38 – 41**)

38. $(x-5)^2 = 81$

39. $(x+7)^2 - 36 = 0$

40. $(3x+2)^2 = 64$

41. $5(x-2)^2 = 10$

Solve the following equations by the quadratic formula. (Express the answers in surd form if necessary.) (**42 – 45**)

42. $x^2 + 17x + 60 = 0$

43. $x^2 - 14x + 49 = 0$

44. $x^2 + 9x + 17 = 0$

45. $x^2 - 2x + 15 = 0$

Consolidation Question

Solve the following equations by the quadratic formula. (Express the answers in surd form if necessary.) (**46 – 49**)

46. $2x^2 + 3x - 4 = 0$

47. $4x^2 + 9x + 3 = 0$

48. $3x^2 - 5x - 3 = 0$

49. $-2x^2 - 7x - 14 = 0$

Non-foundation Topics

Solve the following equations by the method of taking square roots. (Express the answers as surds in their simplest form.) (**50 – 53**)

50. $(x - 8)^2 = 40$

51. $(2x + 3)^2 = 72$

52. $2(2x - 5)^2 = 48$

53. $6(4x + 3)^2 = 108$

Solve the following equations. (Express the answers as surds in their simplest form.) (**54 – 61**)

54. $8 - 3x^2 + 6x = 0$

55. $2x^2 - 7 = 2x$

56. $5x^2 = 10x + 7$

57. $4x^2 - 5 = 6x$

58. $\dfrac{4 - x^2}{2} = x$

59. $\dfrac{x^2}{2} - \dfrac{x}{5} = \dfrac{1}{4}$

60. $1 - 2x^2 = -\dfrac{5}{4}x$

61. $\dfrac{2(1 - x)}{3} = x(2 - x)$

Challenging Question

Solve the following equations. (Express the answers as surds in their simplest form or in the form of $a+bi$ if necessary, where a and b are real numbers.) (**62 – 67**)

62. $2x^2+5x+7=0$

63. $3x^2+4x+8=0$

64. $5x^2+8=0$

65. $\frac{x^2}{3}+\frac{1}{5}=\frac{x}{4}$

66. $\frac{x+4}{3}=-2x(x+1)$

67. $(3-x)(x+3)=\frac{(x+7)(x+9)}{2}$

68. It is given that the equation $x^2+px+q=0$, where p and q are real numbers.

(a) Consider expanding $(x+\frac{p}{2})^2$, prove that $(x+\frac{p}{2})^2=\frac{p^2}{4}-q$.

(b) Prove that $x=\frac{-p\pm\sqrt{p^2-4q}}{2}$.

(c) Hence solve the following equations. (Express the answers as surds in their simplest form or in the form of $a+bi$ if necessary, where a and b are real numbers.)

(i) $x^2+4x-8=0$

(ii) $2x^2-6x+9=0$

End

Build - up Exercise 2C

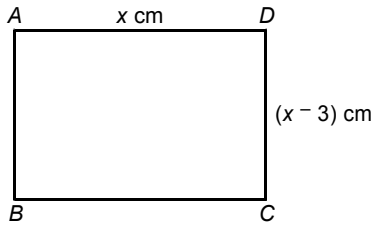
Fundamental Question

69. It is given that the price of a plate with the radius of r cm is $\$(34-9r+r^2)$. Find the radius of the plate if the price is \$44.

70. It is given that the cost of a wardrobe with the height of h m is $\$(500+300h+200h^2)$. Find the height of the wardrobe which costs \$2 500.

71. It is given that $1+2+3+\cdots+n=\dfrac{n(n+1)}{2}$, where n is a positive integer. If $1+2+3+\cdots+n=120$, find the value of n.

72. In the figure, the area of rectangle $ABCD$ is 130 cm^2. Find the value of x.



73. If the sum of the square of y and the square of $y+5$ is 97, find the values of y.

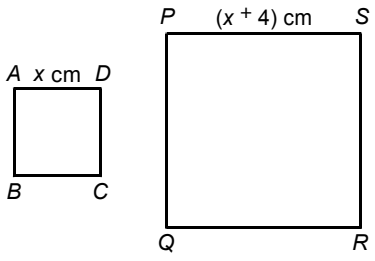
Consolidation Question

74. The product of two consecutive positive odd numbers is 323.

(a) If the smaller number is x, express the larger number in terms of x.

(b) Find the two numbers.

75. In the figure, $ABCD$ and $PQRS$ are squares, and the sum of their areas is 250 cm^2. Find the area of $PQRS$.

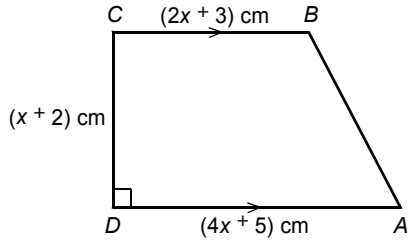


76. The perimeter of a rectangle is 50 cm.

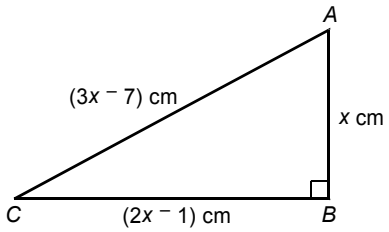
(a) If the length of the rectangle is ℓ cm, express the width of the rectangle in terms of ℓ.

(b) If the area of the rectangle is 154 cm^2, find the length and width of the rectangle.

77. In the figure, $ABCD$ is a trapezium, where $CD \perp AD$. If the area of $ABCD$ is $176\ \text{cm}^2$, find the length of AD.

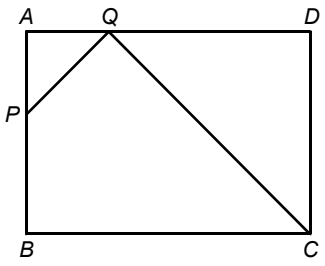


78. In the figure, ABC is a right-angled triangle, where $AB = x$ cm, $BC = (2x - 1)$ cm and $AC = (3x - 7)$ cm. Find the perimeter of ΔABC.



79. In a rectangular coordinate plane, the distance between the two points $(k - 2, -k)$ and $(2k - 3, k)$ is 13. Find the values of k.

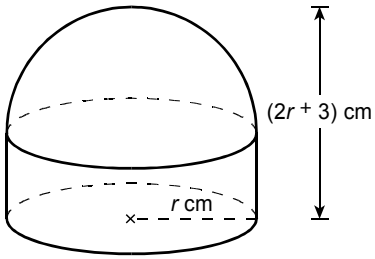
80. In the figure, $ABCD$ is a rectangle. P and Q are the points on AB and AD respectively, where $AP = AQ = x$ cm, $BC = 10$ cm and $CD = 9$ cm.



(a) Express the area of the quadrilateral $BCQP$ in terms of x.

(b) If the area of the quadrilateral $BCQP$ is $50\ \text{cm}^2$, find the values of x.

81. In the figure, a solid consists of a right cylinder and a hemisphere with the radius of r cm. The height of the solid is $(2r + 3)$ cm and the total surface area is 216π cm^2.

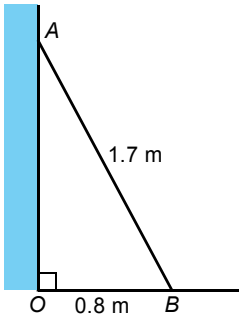


(a) Find the value of r.

(b) Find the total volume of the solid. (Express the answer in terms of π.)

Challenging Question

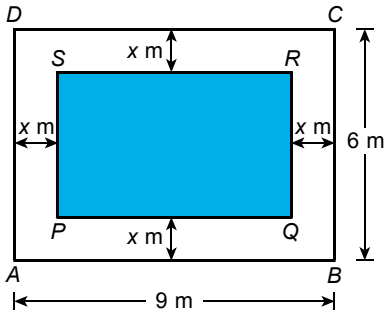
82. In the figure, a pole AB of 1.7 m long leans against a vertical wall OA. The foot of the pole B and the wall are 0.8 m apart.



(a) Find the length of OA.

(b) If the foot of the pole slides x m away from the wall such that the top of the pole slides down by the same distance, find the value of x.

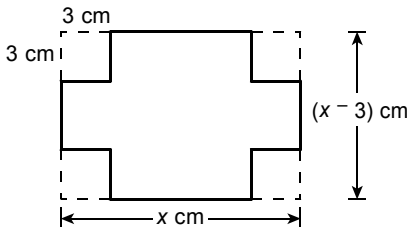
83. In the figure, a rectangular sitting room $ABCD$ is 9 m long and 6 m wide. A rectangular carpet $PQRS$ is placed in the middle of the sitting room such that it is surrounded by a path with a uniform width of x m.



(a) Express the area of the carpet in terms of x.

(b) If the area of the carpet is one third the area of the sitting room, find the length and width of the carpet.

84. A square with sides of 3 cm each is cut from each corner of a rectangular cardboard with the length of x cm and the width of $(x - 3)$ cm. Then the cardboard is folded up to form a box without a lid.



(a) Express the capacity of the box in terms of x.

(b) If the capacity of the box is $375\ \text{cm}^3$, find the value of x.

85. Mandy is 35 years younger than her father and 28 years younger than her mother.

(a) If Mandy is currently x years old, express the ages of her father and mother in terms of x.

(b) After 5 years, the sum of the ages of Mandy's father and mother is the square of Mandy's age. What is the current age of Mandy?

86. The sum of the digits of a two-digit number is 6. Let x be the units digit.

(a) Express the tens digit in terms of x.

(b) Express the value of the number in terms of x.

(c) If the two-digit number is equal to three times the product of its tens and units digits, find the number.

Build - up Exercise 2D

Fundamental Question

87. Find the value of the discriminant of each of the following equations.

(a) $x^2 + 7x + 5 = 0$

(b) $x^2 - 22x + 121 = 0$

(c) $4x^2 = 5x - 7$

(d) $3x^2 + 15 = 5x$

(e) $2x^2 = 3x$

(f) $4x^2 = -27$

88. Determine the nature of roots of each of the following equations.

(a) $x^2 - 8x - 3 = 0$

(b) $4x^2 - 3x + 9 = 0$

(c) $6x^2 + 1 = 0$

(d) $16x^2 - 8x + 1 = 0$

(e) $8 - 3x^2 = 5x$

(f) $20x - 4x^2 = 45$

89. Let k be a constant. Express the discriminant of each of the following equations in terms of k.

(a) $x^2 + 2x + k = 0$

(b) $-2x^2 + 8x + k = 0$

(c) $kx^2 - 3x + 5 = 0$

(d) $2x^2 + 2kx - 3 = 0$

Consolidation Question

90. **(a)** Let k be a constant. Express the discriminant of the equation $3x^2 - 16x - 4k = 0$ in terms of k.

(b) If the equation has real roots, find the range of values of k.

91. **(a)** Let k be a constant. Express the discriminant of the quadratic equation $kx^2 + 12x + 4 = 0$ in terms of k.

(b) If the equation has two equal real roots, find the value of k.

92. **(a)** Let k be a constant. Express the discriminant of the quadratic equation $5kx^2 + 24x + 6 = 0$ in terms of k.

(b) If the equation has no real roots, find the range of values of k.

93. If each of the following quadratic equations has two equal real roots and k is a constant, find the value of k.

(a) $kx^2 - 10x + 1 = 0$

(b) $3x^2 + 8x + 4k = 0$

94. If each of the following quadratic equations has two unequal real roots and k is a constant, find the range of values of k.

(a) $2x^2 - x - k = 0$

(b) $kx^2 - 2x + 4 = 0$

95. If each of the following quadratic equations has no real roots and k is a constant, find the range of values of k.

(a) $2x^2 - 9x + k = 0$

(b) $kx^2 - 4x - 5 = 0$

96. If each of the following quadratic equations has a double real root and k is a constant, find the values of k.

(a) $2x^2 + 6x - (k + 2) = 0$

(b) $(k - 3)x^2 - 3kx + 36 = 0$

97. If each of the following quadratic equations has two distinct real roots and k is a constant, find the range of values of k.

(a) $5x^2 - 6x - 2(k - 1) = 0$

(b) $(3k - 1)x^2 + 4x - 4 = 0$

98. If each of the following quadratic equations has two non-real roots and k is a constant, find the range of values of k.

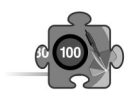
(a) $1 - (k + 2)x^2 = 6x$

(b) $2x(x + 3) = 4x + k$

99. If each of the following quadratic equations has real roots and k is a constant, find the range of values of k.

(a) $3(2x^2 - 1) = 2(4x + k)$

(b) $(k - 1)x^2 - 2kx + (k + 2) = 0$



Challenging Question

100. **(a)** If the quadratic equation $3kx^2 - kx + 2 = 0$ has a double real root and k is a constant, find the value of k.

(b) From the result of **(a)**, solve the equation $3kx^2 - kx + 2 = 0$.

101. **(a)** If the quadratic equation $kx^2 + (3k-1)x + (2k-1) = 0$ has two equal real roots and k is a constant, find the value of k.

(b) From the result of **(a)**, solve the equation $kx^2 + (3k-1)x + (2k-1) = 0$.

102. **(a)** It is given that k is a non-zero constant. Express the discriminant of the quadratic equation $kx^2 + (k-2)x - 2 = 0$ in terms of k.

(b) Prove that the equation has real roots.

103. It is given that $2 = 5x - (k^2 + 4)x^2$ is an equation in x. Prove that the equation has no real roots for all real numbers k.

Build - up Exercise 2E

Fundamental Question

Find a quadratic equation in x with each of the following sets of roots. (**104 – 107**)

104. 2, 9

105. −5, 5

106. $-\frac{1}{3}$, 0

107. a double root $\frac{3}{4}$

Non-foundation Topics

In each of the following, if α and β are the roots of the quadratic equation, find $\alpha + \beta$ and $\alpha\beta$. (**108 – 111**)

108. $x^2 - 2x + 4 = 0$

109. $x^2 + 3x - 7 = 0$

110. $3x^2 - 5x - 9 = 0$

111. $-4x^2 + 2x + 3 = 0$

Consolidation Question

112. Find a quadratic equation in x with roots $4 + \sqrt{3}$ and $4 - \sqrt{3}$.

113. Let m be a constant. If $\frac{3}{2}$ is a root of the equation $4x^2 - mx + 15 = 0$,

(a) find the other root.

(b) find the value of m.

114. Let k be a constant. If the product of roots of the quadratic equation $kx^2 + 5x - (12 - k) = 0$ is -3, find the value of k.

115. If α and β are the roots of the equation $3x^2 + x - 4 = 0$, find the value of each of the following.

(a) $6\alpha + 6\beta$

(b) $(6\alpha)(6\beta)$

116. If α and β are the roots of the equation $-2x^2 + 4x - 5 = 0$, find the value of each of the following.

(a) $\frac{1}{\alpha\beta}$

(b) $\frac{1}{\alpha} + \frac{1}{\beta}$

117. If α and β are the roots of the equation $x^2 + 7x - 5 = 0$, find the value of each of the following.

(a) $\alpha^2 + \beta^2$

(b) $\alpha^3 + \beta^3$

118. If α and β are the roots of the equation $3x^2 - 2x + 8 = 0$, find the value of each of the following.

(a) $(1 + 2\alpha)(1 + 2\beta)$

(b) $(\alpha - \frac{1}{\beta})(\beta - \frac{1}{\alpha})$

119. If α and β are the roots of the equation $8x^2 - 2x + 7 = 0$, find a quadratic equation in x with each of the following sets of roots.

(a) -3α, -3β

(b) $2 - \alpha$, $2 - \beta$

120. If α and β are the roots of the equation $2x^2 + 3 = 3x$, find a quadratic equation in x with each of the following sets of roots.

(a) $\frac{1}{\alpha}, \frac{1}{\beta}$

(b) $\frac{\alpha}{\beta}, \frac{\beta}{\alpha}$

121. If α and β are the roots of the equation $7x^2 - 2x + 6 = 0$, find a quadratic equation in x with each of the following sets of roots.

(a) $2\alpha - 1, 2\beta - 1$

(b) α^2, β^2

122. Let m be a constant. If a root of the equation $5x^2 - (5m + 1)x + m = 0$ is the reciprocal of the other root, find the two roots.

123. Let k be a constant. If a root of the equation $x^2 - 8x + 2k = 0$ is 3 times the other root, find the value of k.

124. Let k be a constant. If α and β are the roots of the equation $3x^2 + 42x + 2k = 0$ and $\alpha : \beta = 4 : 3$, find the value of k.

Challenging Question

125. It is given that α and β are the roots of the equation $x^2 + kx - 2 = 0$, where $k > 0$. If $\alpha^2 + \beta^2 = 13$,

(a) find the value of k.

(b) find a quadratic equation in x with roots α^3 and β^3.

126. It is given that m is a constant and the sum of roots of the equation $10x^2 + 3mx + 2 = 0$ is greater than the product of roots by $\frac{7}{10}$.

(a) Find the value of m.

(b) From the result of **(a)**, solve the equation $10x^2 + 3mx + 2 = 0$.

127. If α and β are the roots of a quadratic equation, where $\alpha + \beta = 4$ and $\alpha\beta = 12$,

(a) prove that the equation has no real roots.

(b) find the complex roots of the equation. (Express the answers as surds in their simplest form if necessary.)

128. Let k be a constant. It is given that α and β are the roots of the equation $x^2 - kx + 3 = 0$.

(a) Prove that $\alpha^2 = k\alpha - 3$.

(b) Express $\alpha^2 + k\beta$ in terms of k.

129. It is given that m and n are distinct real numbers and $\begin{cases} 3m^2 + m - 3 = 0 \\ 3n^2 + n - 3 = 0 \end{cases}$.

(a) Find the values of $m + n$ and mn.

(b) Find the value of $m^2 + n^2$.

(c) Find a quadratic equation in x with roots $\frac{1}{m^2}$ and $\frac{1}{n^2}$.

130. It is given that α and β are the roots of the equation $4x^2 - 3x - 1 = 0$.

(a) Find the value of each of the following.

(i) $\frac{\alpha + \beta}{2}$

(ii) $\frac{\alpha\beta}{4}$

(b) Let m be a constant. Find a quadratic equation in x with roots $\frac{\alpha}{2} - m$ and $\frac{\beta}{2} - m$.

End